

Decentralized Routing and Power Allocation in FDMA Wireless Networks based on H_∞ Fuzzy Control Strategy

Salman Baromand¹, Mohammad A. Nekouie²

¹Department of Electrical Engineering, K. N. Toosi University, Tehran, Iran. Email: S.baromand@ee.kntu.ac.ir

²Department of Electrical Engineering, K. N. Toosi University, Tehran, Iran. Email: manekoui@eetd.kntu.ac.ir

Abstract

Simultaneous routing and resource allocation has been considered in wireless networks for its performance improvement. In this paper we propose a cross-layer optimization framework for worst-case queue length minimization in some type of FDMA based wireless networks, in which the the data routing and the power allocation problem are jointly optimized with Fuzzy distributed H_∞ control strategy . with the presented formulation based on the minimization of the queuing length in each node, the routing and resource allocation problem is formulated as a decentralized fuzzy H_∞ optimal control problem for a wireless mesh network. the presented control strategy determines the transmit power in FDMA systems in which each node has fixed set of powers to be allocated to its outgoing links. Using the proposed control strategy a robust routing performance will be achieved in the presence of unknown network delays in network modeling. also with using fuzzy decision rules in the proposed H_∞ controller strategy, we try to improve the network performance criteria and avoid packet loss in the network.

Keywords: Wireless FDMA network, routing and resource allocation, Decentralized H_∞ control, Fuzzy control.

© 2014 IAUCTB-IJSEE Science. All rights reserved

1. Introduction

In wireless data networks, routing is generally a function of the link capacities which are determined by wireless channel variations and the corresponding allocated radio resources, e.g. power, frequency, and time slots. So the optimal performance of wireless mesh network can achieve with simultaneous resource allocation and routing strategy [1]. Furthermore, due to limitations of different radio resources, efficient resource usage plays an important role in the efficiency and performance of wireless network. In this paper we investigate joint routing and radio resource allocation for optimal system performance. The routing problem is usually implemented based on

different objectives such as achieving the shortest path between the source and destination, minimizing congestion, minimizing end-to-end delay, or controlling the packet loss, see, e.g. [2],[3],[4], [5]. The choice of routing objective is in fact related to the network and service parameters.

In wireless data networks, capacity of data links are not necessarily fixed, and link capacities are determined by the allocated communication resources (e.g., power, frequency, or time slots) among the various links [6].so change in resource allocation will result in link capacity variation, and has influence on the routing decision. In particular, the optimal routing problem in the network layer and resource allocation problem in the radio control layer are coupled through the link capacities, so the

optimal performance can only be achieved by joint optimization of routing and resource allocation [1]. Therefore, recently the joint resource allocation and routing optimization problem has been one of the most intensively studied areas. Solution approaches may be roughly classified as being static, e.g., [1], [7], [8], dynamic, e.g., [9], [10], or quasi-static [6]. In some references such as [1],[7], [8] the joint routing and resource allocation (JRRA) problem is formulated as a convex optimization problem over the network flow variables and the communications resource variables. In wireless networks cross-layer routing and resource allocation is considered for performance improvement. While in existing work usually the delayed dynamic model of network is not considered so routing strategy must be robust with respect to this uncertainty. Therefore, in this paper, we propose a optimization strategy that achieves robust performance under delayed dynamic model of each node. In particular, it achieves optimal performance under a worst-case queue length in each node. Our main focus in this paper is to extend the model in [11], [12], for FDMA wireless networks. Further we propose a distributed fuzzy routing algorithm for joint resource allocation and routing based on the queuing dynamics, where the presented fuzzy routing strategy guaranties both reliability and flexibility in the dynamic routing controller design procedure. In this algorithm each node makes its own power allocation decisions and constructs its own routing tables based on information from its adjacent nodes.

In this paper we focus on wireless FDMA networks in which each node has fixed set of transmit power to be allocated to its outgoing links. Also we propose the H_∞ fuzzy control strategy for routing problem based on minimization of the worst case of queue length. Our methodology is geared towards development of a decentralized routing algorithm for wireless mesh networks. Hence, each node in the network requires only its own local information to route the received messages. The contributions of this paper are the following: *i*) here unlike the related literature, we consider description of the traffic model (network flow model) in terms of the queuing delayed dynamics, *ii*) according to the presented distributed algorithm the computational complexity of the proposed methodology is lower compared to centralized algorithms, *iii*) the proposed cross layer optimization problem presented as a linear stabilization problem of network dynamic model with fix controller in which presented robust fuzzy controller, guarantees the overall network stability and minimizes the worst-case queue length. The

paper is organized as follows. Section II describes the routing problem based on the network dynamic and resource model. According to the presented uncertain network dynamic model and resource model, cross layer optimization problem is presented in section III. For the presented cross layer optimization problem, in section IV new fuzzy control strategy to solve the resource allocation and routing optimization problem is introduced. In section V we implement the proposed algorithm in FDMA wireless networks for power allocation problem and the simulation.

2. Network dynamic Model

2.1 Network Linear dynamic Model

We represent a mesh communication network by a graph $V = (\mathbf{N}, \mathbf{L})$, where \mathbf{N} is the set of n network nodes, and \mathbf{L} is the set of l directed links. Corresponding to each packet a predefined destination has been assigned which is a node in the network. At each node, data packets to be sent to their final destination are subject to several types of delay until they reach their final destinations .

In each node i , corresponding to each destination, d , a queue is considered and at time instant t , $q_i^d(t)$ denotes the number of packets in that queue. Assume that at time instant t , $f_i^d(t)$ is the flow rate of external data packets with destination d entering the network at node i , and $u_{ji}^d(t)$ is the flow rate of messages from node i to j destined to node d .

For $\forall i \in \mathbf{N}$ and $d = 1, 2, \dots, n$, we can define

$$\mathbf{x}_i(t) = \text{vec} \{q_i^d\} \in \mathfrak{R}^n, d = 1, 2, \dots, n. \quad (1)$$

$$\mathbf{u}_i(t) = \text{vec} \{u_{ij}^d\} \in \mathfrak{R}^{l_i}, \mathbf{w}_i(t) = \text{vec} \{f_i^d\} \in \mathfrak{R}^n, \quad (2)$$

where for $\theta_i, i = 1, \dots, n$, $\text{vec} \{\theta^i\} := [\theta^1, \dots, \theta^n]^T$, $\mathbf{x}_i(t)$ is a vector of the queue lengths of packets in node i destined to node d , $d = 1, 2, \dots, n$, $\mathbf{u}_i(t)$ consists of the flows sent from node i to node d through the downstream node with a time delay and l_i is the number of outgoing links from node i . also interactions vector $\mathbf{w}_i(t)$ consists of the flow rate of external data packets entering the network at node i [1], [5], [12].

Based on the above definitions, the queue dynamic model of wireless mesh network in each node is then presented as the following state space model:

$$\dot{\mathbf{x}}_i(t) = B_i \mathbf{u}_i(t) + \sum_{j \in \mathcal{U}_i} B_{dij} \mathbf{u}_j(t - \tau_{ij}) + B_{\omega_i} \mathbf{w}_i(t), \quad (3)$$

Where $\mathcal{U}_i := \{j | \text{There exists a link from } j \text{ to } i\}$. also $B_i \in \mathfrak{R}^{n \times l}$ and $B_{dij} \in \mathfrak{R}^{n \times l}$ represent network connectivity, where B_i (B_{dij}) elements are equal to -1 (1) if node j is a downstream (upstream) neighbor of node i and is zero otherwise [1]. In addition, B_{ω_i}

is equal to an identity matrix and \mathfrak{D}_i assumes as set of downstream neighbors of node i . The unknown differentiable function $\tau_{ij}(t)$, denote the time-varying delays, and considered as the sum of the following delays: transmitting delay (the time between starting and ending the transmission of a packet from node i to node j), propagating delay (the time required for propagating a packet on each link) and processing delay (the time required for each packet to be processed in node i). also time-varying delay $\tau_{ij}(t)$, for all $t \geq 0$ satisfies

$$0 \leq \tau_{ij}(t) \leq d_{ij} < \infty, 0 \leq \dot{\tau}_{ij}(t) \leq \mu_{ij} < 1, \quad (4)$$

where $d_i = \max_j(d_{ij})$, $\mu_i = \max_j(\mu_{ij})$. This mainly justifies assuming $\dot{\tau}_{ij}(t) < 1$ in most practical applications.

2.2. Communication Resource Model

Let \mathbf{h}_i be a vector of communication variables allocated to the links of i^{th} node and \mathbf{h}_{l_i} will be a vector of communication variables associated with link l_i . in this paper we will focus on the case where the link capacity c_{ij} is only a function of the local resources \mathbf{h}_i , i.e., $c_{ij} = \varphi_i(\mathbf{h}_i)$.

In our optimization strategy for Gaussian channel with FDMA, a disjoint bandwidth, W_{ij} and power, P_{ij} are pre-assigned to j^{th} link of node i . The received power at node j is $\delta_{ij}P_{ij}$, where δ_{ij} is the channel gain corresponding to the wireless link (i, j) . The receiving node j is also subject to independent additive white Gaussian noise (AWGNs) with power spectral density N_0^j . The Shannon capacity of link (i, j) is a concave and increasing function of (P_{ij}, W_{ij}) :

$$c_{ij}(P_{ij}, W_{ij}) = W_{ij} \log_2 \left(1 + \frac{\delta_{ij}P_{ij}}{N_0^j W_{ij}} \right), j = 1, \dots, l_i. \quad (5)$$

2.3. Network and communication constraints

2.3.1. Network physical constraints

Assuming a fully connected network, in a network with n nodes, there are $n - 1$ destination nodes. In a wireless mesh network, we need to also consider certain physical characteristics of the wireless networks which impose extra constraints. A typical set of such constraints are as follows:

$$\mathbf{u}_i(t) \geq 0, i = 1, \dots, n, \quad (6)$$

$$\mathbf{x}_i(t) \geq 0, i = 1, \dots, n, \quad (7)$$

$$G_i \mathbf{u}_i(t) \leq c_i(\mathbf{h}_i) \in \mathfrak{R}^{l_i}, \quad (8)$$

$$G_{k_i} \mathbf{u}_i(t) \leq c_{k_i}(\mathbf{h}_{k_i}), i = 1, \dots, n, k_i = 1, \dots, l_i,$$

$$Q_{dij} \mathbf{x}_i(t) \leq x_{maxd_{ij}}, d = 1, \dots, \bar{d}, \quad (9)$$

where c_i is the outgoing links capacity vector of i^{th} node. also c_{k_i} is the link capacity and \mathbf{h}_{k_i} is the vector of communication resources allocated to adjacent link to node i . Furthermore x_{maxd_i} is the buffer size limitation and \bar{d} is the number of destination nodes.

Due to the physical constraints (6)-(7), the queue length at each node and the flow rate of packets in the network must be nonnegative. The capacity constraint in (8) states that the total flow in each link cannot exceed its capacity, c_{k_i} . The last constraint, indicates that to avoid packet loss the length of the queue should always remain smaller than the maximum queue length, $x_{maxd_{ij}}$.

In this model G_{k_i} is defined such that $G_{k_{ij}}$ is equal to 1, if \mathbf{u}_{ij} is a downstream flow of \mathbf{x}_i and has the same destination as the corresponding q_i^d . Therefore, G_{k_i} is defined such that by multiplying G_{k_i} to \mathbf{u}_i , one yields the total flows that should go through the link k_i , and Q_{dij} is defined such that $Q_{dij} \mathbf{x}_i$ leads to the queueing length of the buffer dji , for $d = 1, \dots, \bar{d}$, $i, j = 1, \dots, n$.

2.3.2. Communication constraints

The mentioned communication parameters are themselves limited by various resource constraints, such as limits on the total transmit power at each node or the total signal bandwidth available across the whole network. Generally in each node i , we can use the following generic model to relate the limitation of communications variables \mathbf{h}_i in each node:

$$F_i \mathbf{h}_i(t) \leq g_i \quad (10)$$

$$\mathbf{h}_i(t) \geq 0, \quad (11)$$

The first set of constraints describe resource limits and second constraint specifies that the communications variables are non-negative. In Gaussian channel with FDMA, the transmit power allocated at node $i \in V$ are constrained by the corresponding node powers limits,

$$\sum_{j \in L_U(i)} P_{ij} \leq P_{imax}, P_{ij} \geq 0. \quad (12)$$

where $L_U(i)$ is the set of links that emanate from i . Constraints, (12), indicate the limitation of allocated power of outgoing links in each node should always remain smaller than the maximum power (P_{imax}). Later in this paper we will show that this typical capacity functions for Gaussian channels with FDMA fit into our framework.

3. Joint Congestion Control and Resource Allocation

A model for the wireless mesh network can be obtained by combining the network queue dynamic model(3), the communication model (5), network physical constraints and communication constraints (10) described in the previous section. In wireless data network, the link capacities, among other things, depend on the allocation of communication resources, and the overall optimal performance of the network can be achieved by joint optimization of routing and resource allocation.

3.1. A generic convex optimization formulation

According to the considered network model the optimization problem can be formulated as problem of minimizing a objective function which is considered as the worst-case queuing length due to the external traffic inputs in network nodes.

so by selecting the regulated output as $\mathbf{z}_i(t) = C_i \mathbf{x}_i(t)$ (where C_i can selected as unit matrix), mentioned objective function can be formulated as the problem of minimizing the infinity norm of T_{zw} , i.e., the transfer function matrix from the input vector $\mathbf{w}(t) = \text{vec}\{\mathbf{w}_i(t)\}$ to the output vector $\mathbf{z}(t) = \text{vec}\{\mathbf{z}_i(t)\}$. so with the presented transfer function, we can formulate minimizing worst-case queuing length due to the external inputs, as:

$$\min_{\mathbf{w}} \frac{\|\mathbf{z}(t)\|_2}{\|\mathbf{w}(t)\|_2} = \min \|T_{zw}\|_{\infty}. \quad (13)$$

3.2. Uncertain representation of cross layer optimization problem

In case of wireless data network, one of the common assumptions for constraint (8) is:

$$G_i \mathbf{u}_i(t) = \alpha_i c_i(\mathbf{h}_i), 0 \leq \alpha_i \leq 1, i = 1, \dots, n. \quad (14)$$

Note that, α_i can be either fixed or per-specified design parameters which affect the amount of capacity usage of data links. It is optimal to select $\alpha_i = 1$. In fact, some can say that it is optimal to consume the whole capacity of the data links. It is worth noting that we can consider α_i as designing parameter, in which this parameter can be determined due to the information about the congestion in downstream nodes of i .

without loss of generality in the following, for (14) we can write:

$$\mathbf{u}_i(t) = (G_i^T G_i)^{-1} G_i^T \times \alpha_i c_i(\mathbf{h}_i). \quad (15)$$

Since in (15), \mathbf{h}_i and \mathbf{u}_i are related in a non linear form, without changing the general form or

dynamic equations, we can consider \mathbf{u}_i as a summation of $G_i \times \alpha_i \bar{\mathbf{u}}_i$ and nonlinear term, $\Delta G_i \times \alpha_i \bar{\mathbf{u}}_i$, so that $\bar{\mathbf{u}}_i = \mathbf{h}_i$. In this case the only requirement for ΔG_i is to be bounded. So

$$\mathbf{u}_i(t) = (G_i^T G_i)^{-1} G_i^T \alpha_i c_i(\mathbf{h}_i) = (\bar{G}_i + \Delta \bar{G}_i(\bar{\mathbf{u}}_i, t)) \alpha_i \bar{\mathbf{u}}_i(t), \quad (16)$$

where

$$\bar{\mathbf{u}}_i(t) = \mathbf{h}_{k_i}(t) \in \mathfrak{R}^{l_i \times 1}, k_i = 1, \dots, l_i \quad (17)$$

Attention that $\Delta \bar{G}_i \times \bar{\mathbf{u}}_i(t)$, can be assumed as a continuous matrix function that represents the link capacity estimation error. In section 5 with presenting the change of variable and according to this note that we don't use the estimation in our formulation, we will show that we can assume $\Delta \bar{G}_i = \Delta \bar{G}_j = 0$.

Based on this representation, resource variables \mathbf{h}_{k_i} , which are allocated by the nodes, can be considered as a control variable.

Consider a wireless data network described by the network model (3) and the generic formulation (14)-(17). So at each node of a wireless data network, one can introduce a linear queue length dynamic regarding to the link capacity and total time delay as follows:

$$\begin{aligned} \dot{\mathbf{x}}_i(t) &= B_i \bar{G}_i \alpha_i \bar{\mathbf{u}}_i(t) \\ &+ \sum_{j \in \mathcal{U}_i} B_{dij} \bar{G}_j \alpha_j \bar{\mathbf{u}}_j(t - \tau_{ij}) + B_{\omega i} \mathbf{w}_i(t) \\ &= \bar{B}_i \alpha_i \bar{\mathbf{u}}_i(t) + \sum_{j \in \mathcal{U}_i} \bar{B}_{dij} \alpha_j \bar{\mathbf{u}}_j(t - \tau_{ij}) + B_{\omega i} \mathbf{w}_i(t), \end{aligned} \quad (18)$$

where $\bar{\mathbf{u}}_i(t) = \text{vec}\{\mathbf{h}_{k_i}(t)\}$ is the resource variable that should be determined by the control strategy.

Now according to the considered model for the operation of wireless mesh network (18) and (6)-(11), we can introduce the new generic formulation of the Cross layer Optimization problem:

Problem \mathcal{O}_1 :

$$\begin{aligned} &\min_{\mathbf{h}_i, \forall i=1, \dots, n} \|T_{zw}\|_{\infty}, \\ &s. t. \\ \dot{\mathbf{x}}_i(t) &= \bar{B}_i \alpha_i \bar{\mathbf{u}}_i(t) + B_{\omega i} \mathbf{w}_i(t) + \sum_{j \in \mathcal{U}_i} \bar{B}_{dij} \alpha_j \bar{\mathbf{u}}_j(t - \tau_{ij}), \\ x_i(t) &\geq 0, \\ Q_{dij} x_i(t) &\leq x_{maxdij}, \\ F_i \mathbf{h}_i(t) &\leq \mathbf{g}_i, \\ \mathbf{h}_i(t) &\geq 0, \end{aligned}$$

For $i = 1, \dots, n$. where $\bar{\mathbf{u}}_i(t) = \mathbf{h}_i(t)$. in fact the objective of the routing problem here is, to design a linear local control law, $\mathbf{h}_i(t) = k_i \mathbf{x}_i(t)$, such that it simultaneously, guarantees stability of the overall network traffic model (18) in presence

of time-varying delays and minimizes the presented global objective function in addition to the presented physical network and communication constraints. In continuance with using a fuzzy representation of channel coefficient α_i , we will propose a fuzzy control strategy for proposed network queuing model to improve the performance of network and optimal usage of links capacity in the network.

4. H_∞ Fuzzy Controller Designing

In previous section, with the proposed uncertain queue length model of each node (18), general Cross layer Optimization problem is described. In fact, the flexibility of presented optimization problem \mathcal{O}_1 is achieved by eliminating of link capacity constraint (8) and presented coefficient α_i . With using a fuzzy representation of channel coefficient α_i , we can propose a fuzzy network model to decrease the congestion and optimal usage of links capacity in the network. so in continuance we will propose a fuzzy representation of channel coefficient α_i in optimization problem \mathcal{O}_1 to improve the routing performance and optimal using of the capacity of data links.

4.1. Fuzzy network model

In order to decrease data congestion in the network and optimal use of links capacity, it is reasonable to route data packets through less congested nodes. so with using a fuzzy representation of channel coefficient α_i in (18), we can propose a network fuzzy model and fuzzy control strategy to increase the routing performance in the network and optimal using of the capacity of data links. Based on the proposed method in [13], we can direct messages based on the following fuzzy control rule:

m^{th} control Rule for node i:

IF $x_i(t)$ is M_{m1} and ...and $x_j(t)$ is M_{mp}

Then $u_i(t) = F_m x_i(t) = \alpha_{jm} k_i x_i(t)$, for $i = 1, \dots, N$, $m = 1, \dots, r$ and $j \in \mathcal{D}_i$.

Where M_{mj} is the fuzzy set and r is the number of rules and k_i will be the local memory-less H_∞ control law.

Also $\alpha_{jm} = \text{vec} \{\alpha_{jm}^1, \alpha_{jm}^2, \dots, \alpha_{jm}^d\}$, $j \in \mathcal{D}_i$ are design parameter which affect the matrices B_i , due to congestion information in downstream nodes of i.

It should be noted that at the i^{th} node, the outgoing flow rates to downstream nodes depends on the queue lengths in downstream node $j \in \mathcal{D}_i$. also the entering flow rates from upstream nodes depends on the present queue lengths in node i so α_{im} , affect

the matrices B_{dij} , due to congestion information in upstream nodes of i. In fact, if for node i there is a congested downstream node, it is better to route messages through other downstream nodes and this strategy provides enough time for the congested nodes to evacuate their buffers to appropriate downstream nodes. Therefore with considering the presented fuzzy control rule, we can increase the routing performance and optimal using of the capacity of data links. so according to the considered uncertain model for the operation of wireless mesh network (18) and presented fuzzy control strategy, we will have following close loop fuzzy system model:

$$\begin{aligned} \dot{\mathbf{x}}_i(t) = & \sum_{j \in \mathcal{D}(i)} \sum_{m=1}^r h_m(x_j(t)) \{ \alpha_{jm} \bar{B}_i k_i \mathbf{x}_i(t) \\ & + B_{\omega i} \mathbf{w}_i(t) \} + \sum_{j \in \mathcal{U}_i} \sum_{m=1}^r h_m(x_i(t)) \{ \alpha_{im} \bar{B}_{dij} \mathbf{k}_j \mathbf{x}_j(t - \tau_{ij}) \}. \end{aligned} \quad (21)$$

Where $h_m(\theta) = v_m(\theta) / (\sum_{m=1}^r v_m(\theta))$, $v_m(\theta) = \prod_{j \in \mathcal{D}_i} M_{mj}(x_j)$, $\theta = x_j(t)$, $j \in \mathcal{D}_i$ Where θ_j , M_{mj} and $v_m(\theta)$ are respectively the premise variables, the fuzzy sets and membership function (dependent on $x_j(t)$, $j \in \mathcal{D}_i$) of the i^{th} node with respect to plant rule m .

Moreover, the fuzzy weighting functions $h_m(\theta)$ satisfy $\sum_{m=1}^r h_m(\theta) = 1$. This strategy provides enough time for the congested nodes to evacuate their buffers to appropriate downstream nodes. also accordingly at the i^{th} node there is knowledge about the rate of the entering messages from the upstream nodes.

4.2. Fuzzy Controller designing

Presented close loop network fuzzy model (21) is a more general representation of (18) and the flexibility of this proposed model is achieved by the matrices α_{jm} and α_{im} . So according to the considered fuzzy model for the operation of wireless mesh network (21) and generic formulation \mathcal{O}_1 , we can introduce the new fuzzy formulation of the Cross layer Optimization problem:

Problem \mathcal{O}_2 :

$$\min_{k_i, i=1, \dots, n} \| T_{zw} \|_\infty,$$

s. t.

$$\begin{aligned} \dot{\mathbf{x}}_i(t) = & \sum_{j \in \mathcal{D}(i)} \sum_{m=1}^r h_m(x_j(t)) \{ \alpha_{jm} \bar{B}_i k_i \mathbf{x}_i(t) + B_{\omega i} \mathbf{w}_i(t) \} \\ & + \sum_{j \in \mathcal{U}_i} \sum_{m=1}^r h_m(x_i(t)) \{ \alpha_{im} \bar{B}_{dij} \mathbf{k}_j \mathbf{x}_j(t - \tau_{ij}) \} \end{aligned}$$

$$\begin{aligned} x_i(t) &\geq 0, \\ Q_{aij}x_i(t) &\leq x_{maxdij}, \\ F_i\dot{h}_i(t) &\leq g_i, \\ \dot{h}_i(t) &\geq 0, \end{aligned}$$

For $i = 1, \dots, n$, where $\dot{h}_i(t) = \sum_{m=1}^r h_m(x_j(t))\alpha_{jm}k_i\mathbf{x}_i(t), j \in \mathcal{D}_i$.

Thus we can present simultaneous resource allocation and routing optimization problem over the presented node-based wireless data networks, as a stabilizing problem of system (21), where resource variables $\dot{h}_i(P_{ij})$ act as control variable. also H_∞ control design strategy is suitable framework for uncertain delayed system such as (21) [9],[18].

Therefore, according to the presented optimization problem \mathcal{O}_2 , the objective of the routing problem here is, to design a fuzzy H_∞ control law, k_i , such that it simultaneously, guarantees stability of the overall network traffic model (21) in presence of time-varying delays and minimizes a global objective function which is considered as the worst-case queuing length due to the external traffic inputs, according to the modelling uncertainties and constraints of problem \mathcal{O}_2 .

In fact with eliminating of link capacity constraint (9) from primal network model and considering fuzzy coefficients α_{im} , we can improve the flexibility of primal network model and it can reduce the conservativeness of control strategy in previous works [1],[4],[12],[16].

In the following we derive a sufficient condition for system represented by (21) to be stabilizable via controller matrix k_i , based on Lyapunov's functional method.

In problem \mathcal{O}_2 , according to the presented fuzzy dynamic close loop model, minimizing the infinity norm of T_{zw} is equivalent to the following optimization problem for represented closed-loop fuzzy system (21)[15]:

$$\begin{aligned} \min \gamma, \\ \text{s. t.} \end{aligned} \tag{22}$$

$$J(\mathbf{w}) < 0,$$

$$J(\mathbf{w}) = \int_0^\infty (\mathbf{z}^T(t)\mathbf{z}(t) - \gamma^2\mathbf{w}^T(t)\mathbf{w}(t))dt, \gamma > 0.$$

Above optimization minimizes the worst-case queuing length, so congestion and the packet loss probabilities simultaneously will be reduced. H_∞ performance, indicated by (22) is satisfied, if the following Hamiltonian function is negative definite [15]:

$$J_H = \frac{dV}{dt} + \mathbf{z}^T\mathbf{z} - \gamma\mathbf{w}^T\mathbf{w}, \tag{23}$$

where $V(\cdot)$ is a Lyapunov-Krasovskii functional such that $V(0) = 0$.

Here, to solve the H_∞ control of the routing problem, Lyapunov-Krasovskii functional is as follows

$$V = \sum_{i=1}^n V_i(\mathbf{x}_i, t) = \sum_{i=1}^n [V_{i0}(\mathbf{x}_i, t) + V_{i1}(\mathbf{x}_i, t) + V_{i2}(\mathbf{x}_i, t)], \tag{24}$$

where

$$V_{i0}(\mathbf{x}_i, t) = \mathbf{x}_i^T(t)P_i\mathbf{x}_i(t),$$

$$V_{i1}(\mathbf{x}_i, t) = 2 \sum_{j \in \mathcal{U}_i} \int_{t-d_{ij}}^t (d_{ij} - t + s)\dot{\mathbf{x}}_j^T(s)R_j\dot{\mathbf{x}}_j ds$$

$$V_{i2}(\mathbf{x}_i, t) = \sum_{j \in \mathcal{U}_i} \int_{t-\tau_{ij}}^t \mathbf{x}_j^T(s)Z_j\mathbf{x}_j ds.$$

and n is the number of nodes in the network, R_j, Z_j and P_i are symmetric positive definite matrices. with this lyapunov function we can present sufficient conditions for the closed-loop stability of (21) as following theorem.

Theorem 1: Consider a wireless traffic network with variable destination nodes whose dynamics is governed by (21), the state feedback routing controller gain k_i guarantee that the closed-loop system is internally stable and $J(w) < 0$, if there exist matrices M_i , nonsingular matrices Y_i , and symmetric positive definite matrices R_j, Z_i , for $i = 1, \dots, n, j \in \mathcal{U}_i$ such that the following LMI condition is satisfied:

$$\begin{aligned} W_{i1} = & \\ \left[\begin{array}{cccccccc} \Omega_{i1} & 0 & B_{\omega_i} & Y_i^T C_i^T & 0 & \Omega_{i5} & \Omega_{i7} & 0 & 0 \\ * & \Omega_{i2} & 0 & 0 & 0 & 0 & 0 & \Omega_{i9} & \Omega_{i11} \\ * & * & -\gamma I & 0 & 0 & \Omega_{i3} & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{i4} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{i6} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{i8} & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{i10} & 0 \\ * & * & * & * & * & * & * & * & \Omega_{i12} \end{array} \right] < 0, \tag{25} \end{aligned}$$

where

$$\Omega_{i1} = M_i^T \alpha_{im}^T \bar{B}_i^T + \bar{B}_i \alpha_{im} M_i + 2n_i I + n_i Z_i,$$

$$\Omega_{i2} = -diag_j\{(1 - \mu_{ij})Z_j\}, j \in \mathcal{U}_i,$$

$$\Omega_{i3} = \sqrt{2n_i d_i} B_{\omega_i}^T,$$

$$\Omega_{i4} = -R_i$$

$$\Omega_{i5} = 0, \Omega_{i6} = -\varepsilon_i^{-1} I, \Omega_{i7} = 0,$$

$$\Omega_{i8} = -(\sqrt{2n_i d_i} \varepsilon_i)^{-1} I,$$

$$\Omega_{i9} = diag_j\{(B_{dij} \alpha_{im} M_j)^T\},$$

$$\Omega_{i10} = -I,$$

$$\Omega_{i11} = 0, \Omega_{i12} = -diag_j\{(\sqrt{2n_i d_i} \varepsilon_i + \rho_i^{-1})^{-1} I\}, j \in \mathcal{U}_i.$$

and $\alpha_{im}, m = 1, \dots, r$ are pre specified designing parameters, and $*$ denotes the entries implied by the symmetry, also $n_i =$ number of downstream node

for i . Moreover The decentralized state feedback controller gain is given by $k_i = M_i Y_i^{-1}$.

Proof: See Appendix I.

5. Cross layer power allocation and routing optimization problem

In this section we will show that the resulting decentralized routing control schemes formally achieve the desired specifications and requirements of Gaussian broadcast channels with FDMA and the joint power allocation and routing problem can be formulated as stabilizing problem.

5.1. Formulation of the cross layer power allocation and routing problem

Consider a wireless data network where each node uses the Gaussian broadcast channel with FDMA to transmit packets over its outgoing links. Here the communication variables are the powers P_i , limited by separate or total power constraints.

For Transmit power allocation we assume that the bandwidth allocation is fixed (unit bandwidth is assigned to each link). We are free to adjust the transmit powers $P_i = \text{vec}_j\{P_{ij}\}$, where P_{ij} , allocated to each link (i, j) , but we impose a total power constraint for the outgoing links of each node (12). Combining the network link capacity constraint (5), and the equation (15), we have:

$$\mathbf{u}_i(t) = (G_i^T G_i)^{-1} G_i^T \times \alpha_i c_i, \quad (26)$$

where

$$c_i = \text{vec}\{c_{ij}\}_j = \text{vec}\left\{W_{ij} \log_2\left(1 + \frac{\delta_{ij} P_{ij}}{N_0^j W_{ij}}\right)\right\}_j, \quad (27)$$

for $j = 1, \dots, l_i$.

Changing the variables

$$\tilde{\mathbf{u}}_{ij} = \log_2\left(1 + \frac{\delta_{ij} P_{ij}}{N_0^j W_{ij}}\right), j = 1, \dots, l_i, \quad (28)$$

we then get

$$c_i = \begin{bmatrix} W_{i1} & 0 & \dots & 0 \\ 0 & W_{i2} & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & W_{il_i} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}}_{i1} \\ \tilde{\mathbf{u}}_{i2} \\ \vdots \\ \tilde{\mathbf{u}}_{il_i} \end{bmatrix}, \quad (29)$$

and with the proposed change of variables for (26), $i = 1, \dots, n, j = 1, \dots, l_i$ we have:

$$\mathbf{u}_i(t) = G_i \alpha_i \tilde{\mathbf{u}}_i(t), \quad (30)$$

where

$$\bar{G}_i = (G_i^T G_i)^{-1} G_i^T \times \text{diag}_j(W_{ij}) \quad (31)$$

$$\tilde{\mathbf{u}}_i(t) = \text{vec}_j\{\tilde{\mathbf{u}}_{ij}(t)\} \in \mathfrak{R}^{l_i \times 1}. \quad (32)$$

As a result, with the presented capacity formula and the change of variables $\tilde{\mathbf{u}}_i(t)$, the cross

layer optimization problem \mathcal{O}_2 can be formulated as following optimization problem:

$$\min_{k_i, i=1, \dots, n} \|T_{zw}\|_{\infty}, \quad (33)$$

s. t.

$$\dot{\mathbf{x}}_i(t) = \sum_{j \in \mathfrak{D}(i)} \sum_{m=1}^r h_m(x_j(t)) \{\alpha_{jm} \bar{B}_i \tilde{\mathbf{u}}_i(t) + B_{\omega i} \mathbf{w}_i(t)\} + \sum_{j \in \mathfrak{U}(i)} \sum_{m=1}^r h_m(x_i(t)) \{\alpha_{im} \bar{B}_{dij} \tilde{\mathbf{u}}_j(t - \tau_{ij})\}$$

$$\mathbf{x}_i \geq 0, Q_{dij} \mathbf{x}_i(t) \leq x_{max\,dij},$$

$$\sum_{j \in L_U(i)} P_{ij} \leq P_{imax},$$

$$\tilde{\mathbf{u}}_i(t) = \sum_{m=1}^r h_m(x_j(t)) k_i \mathbf{x}_i(t)$$

Where

$$\tilde{\mathbf{u}}_{ij} = \log_2\left(1 + \frac{\delta_{ij} P_{ij}}{N_0^j W_{ij}}\right),$$

$$\bar{B}_i = B_i \bar{G}_i = B_i (G_i^T G_i)^{-1} G_i^T \times \text{diag}\{W_{ij}\}$$

$$\text{and } \bar{B}_{dij} = B_{dij} \bar{G}_j.$$

According to this note that we don't use the estimation in our formulation, with the presented changing of variable we can assume $\Delta \bar{G}_i = \Delta \bar{G}_j = 0$.

Now, the objective is to design a fixed linear state feedback control law $\tilde{\mathbf{u}}_i(t) = \sum_{m=1}^r h_m(x_j(t)) k_i \mathbf{x}_i(t)$, and coefficient α_i . According to the control signal $\tilde{\mathbf{u}}_i(t)$ and equation (28) the resource vector $\bar{\mathbf{u}}_i(t) = \text{vec}_j\{P_{ij}\}$ will be determined by:

$$\bar{\mathbf{u}}_{ij}(t) = P_{ij} = (2^{\tilde{\mathbf{u}}_{ij}} - 1) N_0^j W_{ij} / \delta_{ij} \geq 0, \quad (34)$$

Therefore, $\bar{\mathbf{u}}_i(t) = \text{vec}_j\{P_{ij}\}$ is the power fraction that is allocated to link (i, j) by node i .

Note that bandwidths W_{ij} are fix so B_i and B_{dij} are known and fix matrices.

5.2. Network and Resource Constraints

There are various constraints in a network which affect its performance and the corresponding control systems. Therefore, these constraints should be modeled and considered in the controller design. Some of these constraints are considered by [12]. In this paper, we employ the LMI constraints similar to that of in [1].

5.2.1. Buffer Size Limitation

The queue length at each node must not exceed the size of the buffer, therefore the constraint on the

queue buffer size for each subsystem can be defined as follows

$$Q_{dij}x_i < x_{max_{dij}}, i = 1, \dots, n, d = 1, \dots, \bar{d}, \quad (35)$$

where $x_{max_{dij}}$ is the maximum buffer size, and Q_{dij} in each node should be defined such that $Q_{dij}x_i$ shows the queue length corresponding to the packets destined to the same node. We consider $\sum_i = \{x_i(t)|x_i(t)Y_i^{-1}x_i(t) \leq \lambda_i, Y_i^T = Y_i > 0\}$ as the ellipsoid for a selected $\lambda_i > 0$.

By applying invariant set method, and performing some straightforward mathematical manipulations the constraints in (35) can be expressed by the following LMI:

$$W_{i2} = \begin{bmatrix} Y_i & * \\ Q_{dij} & x_{max_{dij}}^2/\lambda_i \end{bmatrix} \geq 0 \quad (36)$$

5.2.2 The Non-negative Orthant Stability

Using this definition, the non-negativity constraint (6)-(7) can drive from the non-negative Orthant stability condition given in [11] and according to the model uncertainty, the presented LMIs are uncertain, so with assuming:

$$(B_i + \Delta B_i)_{sr} \geq (\varphi_i)_{sr}, (B_{dij} + \Delta B_{dij})_{sr} \geq (\theta_{dij})_{sr}, \quad (37)$$

where $(\varphi_i)_{sr}$ and $(\theta_{dij})_{sr}$ are specified parameters, non-negativity constraint can be expressed through the following LMIs:

$$W_{i3} = (\varphi_i M_i)_{sr} \geq 0, s \neq r, \quad (38)$$

$$W_{i4} = (\theta_{dij} M_j) \geq 0, s, r = 1, \dots, n, \quad (39)$$

which satisfies the non-negativity constraint $x_i \geq 0$.

Also by noting that Y_i is a diagonal positive definite matrix, $u_i \geq 0$ is satisfied if the following LMI holds:

$$W_{i5} = (M_i)_{sr} \geq 0, \quad s = 1, \dots, l(n-1), r = 1, \dots, n(n-1). \quad (40)$$

5.2.3. Resource Constraints

For power allocation problem, for presented resource limitation we have:

$$\sum_{j \in L_{U(i)}} \bar{u}_{ij} = \sum_{j \in L_{U(i)}} P_{ij} \leq P_{imax}, \quad (41)$$

according to the above inequality and change of variable formula (28) we can write :

$$F_i \tilde{u}_i \leq \tilde{P}_{imax}, \tilde{u}_i = \text{vec}\{\tilde{u}_{ij}\}, \quad (42)$$

where

$$\tilde{P}_{imax} = (P_{imax}/S_i + l_i) + \sum_{j \in L_{U(i)}} 2^{\hat{u}_{ij}}(1 + \ln 2)\hat{u}_{ij}$$

$$\text{and } F_i = \text{vec}_j\{2^{\hat{u}_{ij}}(1 + \ln 2)\}, S_i = \min_j\{\frac{N_0^j W_{ij}}{\delta_{ij}}\}$$

$$\text{and } \hat{u}_{ij} = \log_2(1 + \frac{\delta_{ij} P_{imax}/l_i}{N_0^j W_{ij}}).$$

See Appendix II.

Inequality (42) can be conservative for large resource limitation but ensure the resource limitation constraint in (12).

According to the (42) and following the same line of argument as used for the capacity constraint in [1], for power allocation problem, we can present limitation constraints (42), as following LMIs:

$$W_{i6} = \begin{bmatrix} Y_i & * \\ F_i M_i & \tilde{P}_{imax}^2/\lambda_i \end{bmatrix} \geq 0, i = 1, \dots, n, \quad (43)$$

where $F_i = \text{vec}_j\{2^{\hat{u}_{ij}}(1 + \ln 2)\}$.

As long as the upper bound limitation of the allocated power with the presented control strategy is within the estimation range, the LMI constraints and controller do not need to be recomputed. attention that we have presented controller designing algorithm, so our control strategy guarantees the overall network stability and the worst-case performance in theory for bounded estimation errors of network dynamic modeling and network constraints.

The LMI presentation of network and communication resource constraints as well as the network model in the previous sections allow us to formulate joint routing and resource allocation optimization problem as a convex LMI optimization problem.

Considering the presented physical and resource constraints as well as the results of Theorem 1, we can conclude that a decentralized H_∞ fuzzy routing controller for the cross layer optimization problem \mathcal{O}_2 (or optimization problem (33)) can be designed by solving the following optimization problem :

Main optimization Problem \mathcal{O}_3 :

$$\min_{M_i, Y_i, Z_i, R_i} \gamma, \quad (44)$$

subject to the LMI constraints, $W_{li}, (l = 1, \dots, 6, i = 1, \dots, n)$.

where W_{l1} satisfies the H_∞ cost function of \mathcal{O}_2 and LMIs $W_{li}, (l = 2, \dots, 6, i = 1, \dots, n)$ satisfy the constraints of problem \mathcal{O}_2 . The proposed algorithm for joint resource allocation and routing is summarized as follows:

Given: network $V = (N, L)$ and node resource set \hat{h}_i ,

Step 1: Formulate the wireless data network parameters and flow model as uncertain delayed system, as in (18) and (26)-(33),

Step 2: determine the physical constraint and resource limitation as LMI, i.e. $W_{li}, (l = 2, \dots, 6)$,

Step 3: determine LMI W_{l1} (4.2) according to the known parameters of dynamic system (21),

Step 4: solve the optimization problem (44) and obtain the corresponding feedback gain, k_i for pre specified α_{im} .

Step 5: with the computed feedback gain, k_i , in step 4 for each node i ($i = 1, \dots, n$), the fuzzy controller signal $\tilde{\mathbf{u}}_i(t)$ obtains from $u_i(t) = \sum_{m=1}^r h_m(x_j(t))k_i x_i(t), j \in \mathcal{D}_i$.

Step 6: Finally the resource variables $u_i(t)$, that should be allocate to the link (i, j) by node i , will be determined according to the controller signal $u_i(t)$ and equations (34).

Note that by stabilizing the queue dynamic in (21), the queue length in nodes, will be minimized. Therefore, the presented control strategy will provide a distributed mechanism to minimize the queue lengths in the presented wireless network nodes, where $\tilde{\mathbf{u}}_i(t)$ is the definite function of resources that allocates to link (i, j) by node i . In fact in the presented control strategy, the resource parameters are control variables that should be determined.

6. Simulation Results

In this section, simulation results are presented to evaluate the performance of our proposed routing control strategy and we will only illustrate how the Gaussian channels with FDMA can fit into this framework.

Consider the sample network selected as a basis, shown in Fig. 1, which has 10 nodes and 20 directed links.

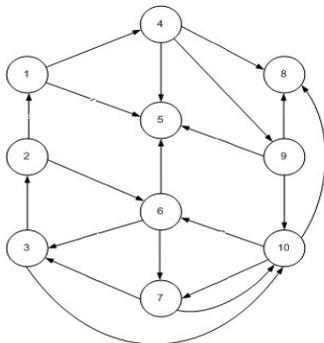


Fig.1. Considered network topology in the simulations.

The destination nodes are selected to be 5 and 8. Therefore, nodes 5 and 8 do not route any messages and are considered as a sink. Consequently, except

nodes 5 and 8, each node has two states: the first state is the queueing length associated with the destination node 5, and the second state is the queueing length associated with the destination node 8. Assume that maximum buffer size of nodes is 1 Kbit and the delay function is taken as a fast varying time structured function $3 + 2\sin(5t)$ sec. Note that as far as the controller is concerned the delay information is considered to be unknown. The external input traffic load for each node is assumed as follows:

$$\mathbf{w}_i = \begin{cases} \mathcal{B} + \mathcal{L} \text{ Kbit/sec} & 1 < t < 3 \text{ sec} , \\ \mathcal{B} & \text{otherwise} , \end{cases} \quad (45)$$

where in this traffic load, parameter \mathcal{B} is considered as the Poisson distribution with the rate of 0.5 Kbit per second. Also assume that in addition the input packet rate \mathcal{B} , the input packets with a flow rate $\mathcal{L} = 0.6$ Kbit/sec, enters to the nodes from the outside of the network at $1 < t < 3$ sec.

Example 1: first we implement the proposed algorithm to minimize the overall data queue length in a FDMA wireless network. For data communication over link (i, j) , the noise power σ_l at each receiver is uniformly distributed on $[0.01, 0.1]$. We also assume $N_0 = 0.1$. We adjust the transmit powers P_{li} allocated to each link, where $P_{imax} = 40, W_i = 1, i = 1, \dots, 8$. Now we use our proposed decentralized routing algorithm based on minimization of the queue length at each node. where the membership functions are demonstrated in Fig.2, and $\alpha_{i1}^1 = \alpha_{i1}^2 = 1, \alpha_{i2}^1 = \alpha_{i2}^2 = 0.7$. Selecting these values for $\alpha_{im}, m = 1, 2$ shows that when one of the downstream nodes is near congestion the router should send fewer messages to that node.

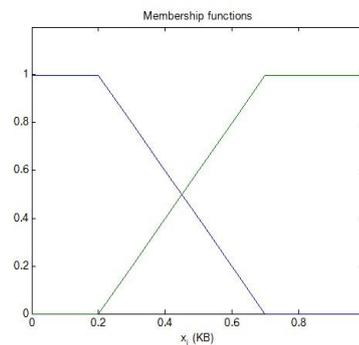
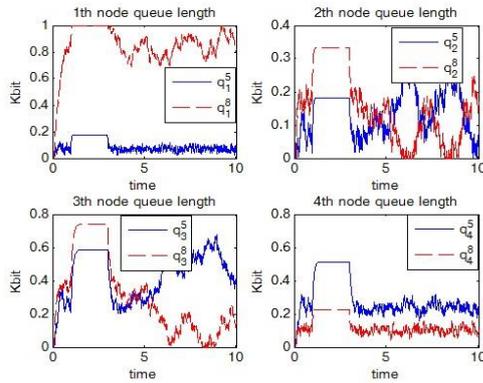
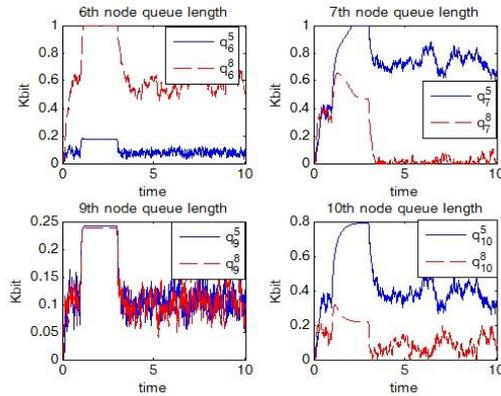


Fig.2. Membership functions of two rule controller

The queueing length of a node in network can be considered as an important issue for evaluating the performance of a routing algorithm. Fig. 3- 4 depicts the queueing lengths of mesh network nodes that are obtained by using our proposed H_∞ routing control algorithm.

Fig.3.Queue lengths q_1, q_2, q_3 and q_4 for versus time.Fig.4. Queue lengths q_6, q_7, q_9 and q_{10} for versus time.

Queue dynamic shows that the represented approach gives needed flexibility to the network to find the efficient route in the network without increasing the congestion of the packets in the downstream nodes. For this examination according to the buffer limit assumption and queue length figures we only have packet loss in node 1, 6 and 7. Also for instance, Fig. 5- 6 demonstrates the transmit powers that allocated to outgoing links of network nodes.

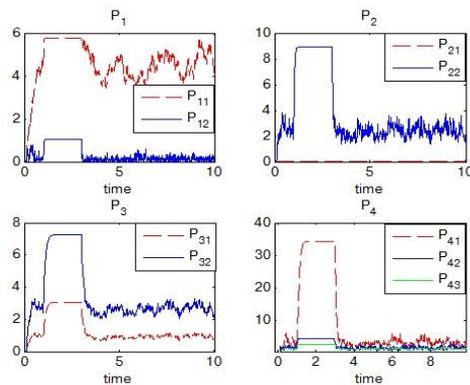


Fig.5. Transmit powers that allocated, with the nodes 1, 2, 3 and 4 to their outgoing links.

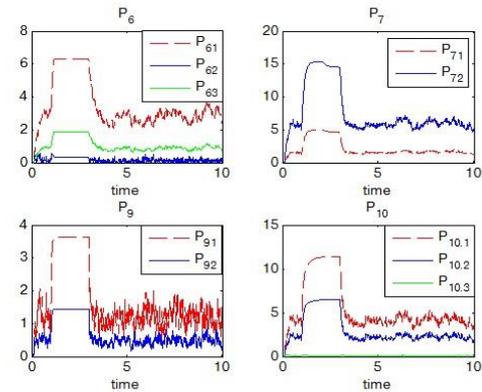


Fig.6. Transmit powers that allocated, with the nodes 6, 7, 9 and 10 to their outgoing links.

7. Conclusion

In this paper, we have developed an optimal distributed algorithm for joint resource allocation and routing for power and bandwidth allocation in FDMA wireless networks. The network routing problem and the resource allocation problem interact through the capacity constraints on the total traffics supported on individual communication links. Using the proposed control strategy a robust routing performance achieved in the presence of unknown network delays and with using fuzzy decision rules in the proposed H_∞ controller strategy, we improve the network performance criteria and avoid packet loss in the network.

Appendix I

To achieve the H_∞ objective function (22), one should show $J_H = \dot{V}(x_i, t) + z^T z - \gamma w w^T < 0$. Taking the time-derivative of V in (24) along the system trajectories in (21), and then substituting that in (23) we will have:

$$\begin{aligned}
 J_H \leq & \sum_{i=1}^n \left\{ \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) \mathbf{x}_i(t)^T (\alpha_{jm}^T \mathbf{k}_i^T \bar{B}_i^T P_i \right. \\
 & \left. + P_i \bar{B}_i \alpha_{jm} \mathbf{k}_i) \mathbf{x}_i(t) \right. \\
 & + \sum_{j \in \mathcal{U}_i} \sum_{m=1}^r h_m(x_i(t)) [\mathbf{x}_i(t)^T P_i \bar{B}_i d_{ij} \alpha_{im} \mathbf{k}_j \mathbf{x}_j(t - \tau_{ij}(t)) \\
 & \left. + \mathbf{x}_j^T(t - \tau_{ij}(t)) \alpha_{im}^T \mathbf{k}_j^T \bar{B}_i^T P_i \mathbf{x}_i(t)] \right. \\
 & + \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) [\mathbf{x}_i^T(t) P_i B_{\omega_i} \mathbf{w}_i(t) \\
 & \left. + \mathbf{w}_i^T(t) B_{\omega_i}^T P_i \mathbf{x}_i(t)] \right. \\
 & + \sum_{j \in \mathcal{U}_i} [2d_{ij} \dot{\mathbf{x}}_j^T(t) R_j \dot{\mathbf{x}}_j(t) - 2 \int_{t-\tau_{ij}}^t \dot{\mathbf{x}}_j^T(s) R_j \dot{\mathbf{x}}_j(s) ds \\
 & + \mathbf{x}_j^T(t) Z_j \mathbf{x}_j(t) - (1 - \mu_{ij}) \mathbf{x}_j^T(t - \tau_{ij}) Z_j \mathbf{x}_j(t - \tau_{ij})] \\
 & \left. + \mathbf{x}_i^T(t) C_i^T C_i \mathbf{x}_i(t) - \gamma \mathbf{w}_i(t)^T \mathbf{w}_i(t) \right\}.
 \end{aligned}$$

Also using the fact that $\sum_{i=1}^n \sum_{j \in \mathcal{U}_i} \mathbf{x}_j(t)^T Q_j \mathbf{x}(t) = \sum_{i=1}^n n_i \mathbf{x}_i(t)^T Q_i \mathbf{x}_i(t)$ [1], where n_i is the number of downstream nodes corresponding to node i , and the fact that $\sum_{l=1}^r h_l(\theta) \Gamma = \Gamma$, and according to the matrix inequalities presented lemmas in [17] we have:

$$\begin{aligned} J \leq & \sum_{i=1}^n \left\{ \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) \mathbf{x}_i(t)^T [k_i^T \alpha_{jm}^T \bar{B}_i^T P_i \right. \\ & \left. + P_i \bar{B}_i \alpha_{jm} k_i \right. \\ & \left. + 2 \sum_{j \in \mathcal{U}_i} \sum_{m=1}^r h_m(x_i(t)) [\mathbf{x}_j(t) \right. \\ & \left. - \tau_{ij}(t)^T \{(\bar{B} d_{ij} \alpha_{im} k_j)^T (\bar{B} d_{ij} \alpha_{im} k_j)\} \right. \\ & \left. \mathbf{x}_j(t - \tau_{ij}(t)) + \mathbf{x}_i^T(t) P_i P_i \mathbf{x}_i(t) + \right. \\ & \left. \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) [\mathbf{x}_i^T(t) P_i B_{\omega_i} \mathbf{w}_i(t) + \right. \\ & \left. \mathbf{w}_i^T(t) B_{\omega_i}^T P_i \mathbf{x}_i(t)] \right. \\ & \left. + \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) [2d_{ij} n_i \dot{\mathbf{x}}_i^T(t) R_i \dot{\mathbf{x}}_i(t)] \right. \\ & \left. + \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) [n_i \mathbf{x}_i^T(t) Z_i \mathbf{x}_i(t)] \right. \\ & \left. - 2 \sum_{j \in \mathcal{U}_i} \sum_{m=1}^r h_m(x_i(t)) \int_{t-\tau_{ij}}^t \dot{\mathbf{x}}_j^T(s) R_j \dot{\mathbf{x}}_j(s) ds \right. \\ & \left. - \sum_{j \in \mathcal{U}_i} \sum_{m=1}^r h_m(x_i(t)) (1 - \mu_{ij}) \mathbf{x}_j^T(t - \tau_{ij}) Z_j \mathbf{x}_j(t - \tau_{ij}) \right. \\ & \left. + \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) [\mathbf{x}_i^T(t) C_i^T C_i \mathbf{x}_i(t) + \right. \\ & \left. \gamma \mathbf{w}_i(t)^T \mathbf{w}_i(t)] \right\} \end{aligned}$$

According to the fact that $\mathbf{x}_i^T(t) P_i P_i \mathbf{x}_i(t) = \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) \{n_i^{-1} \mathbf{x}_i^T(t) P_i P_i \mathbf{x}_i(t)\}$, in above inequality we have

$$\begin{aligned} & \sum_{i=1}^n \sum_{j \in \mathcal{U}_i} \sum_{m=1}^r h_m(x_i(t)) \mathbf{x}_i^T(t) P_i P_i \mathbf{x}_i(t) \quad (15) \\ & = \sum_{i=1}^n \sum_{j \in \mathcal{D}_i} \sum_{m=1}^r h_m(x_j(t)) \mathbf{x}_i^T(t) P_i P_i \mathbf{x}_i(t). \end{aligned}$$

For simplicity we can Assume that $\alpha_{im} = \alpha_{jm}, j \in \mathcal{D}_i$. in fact we assume that similar coefficient α_{im} for all of the nodes. with following some manipulation to make the bilinear matrix inequalities, we will have $J < 0$ if flowing inequality holds:

$$\bar{W}_i = \begin{bmatrix} \theta_{i1} & 0 & \theta_{i2} & C_i^T & \theta_{i3} \\ * & \theta_{i4} & 0 & 0 & \theta_{i5} \\ * & * & -\gamma I & 0 & \theta_{i7} \\ * & * & * & -I & 0 \\ * & * & * & * & \theta_{i8} \end{bmatrix} < 0, \quad (47)$$

where

$$\theta_{i1} = [k_i^T \alpha_{im}^T \bar{B}_i^T P_i + P_i \bar{B}_i \alpha_{im} k_i + 2n_i P_i P_i + n_i Z_i,$$

$$\begin{aligned} \theta_{i2} &= P_i B_{\omega_i}, \theta_{i3} = \sqrt{2n_i d_i} k_i^T \alpha_{im}^T \bar{B}_i^T, \\ \theta_{i4} &= \text{diag}_j \{ (\bar{B} d_{ij} \alpha_{im} k_j)^T (\bar{B} d_{ij} \alpha_{im} k_j) \\ & \quad - (1 - \mu_{ij}) Z_j \}, j \in \mathcal{U}_i \\ \theta_{i5} &= \sqrt{2n_i d_i} \text{vec}_j \{ (k_j^T \alpha_{im}^T \bar{B} d_{ij}^T) \}, \\ \theta_{i7} &= \sqrt{2n_i d_i} B_{\omega_i}^T, \theta_{i8} = -R_i. \end{aligned}$$

Therefore, according to \bar{W}_i we will have $J < 0$ if the above matrix inequality condition hold.

Above inequality is not LMI, so by applying the Schur complement and defining $Y_i = P_i^{-1}$ and $M_i = k_i Y_i$, $R_i = Y_i R_i Y_i^T$, $Z_i = Y_i Z_i Y_i^T$, LMI condition (4.2) will be obtained. The obtained LMI condition guarantees $\dot{V} < 0$, also guarantees that the Hamiltonian J in (22), and consequently J in (22) is negative definite.

Appendix II

In power allocation problem, according to the (28) and resource constraint (12) we have:

$$\begin{aligned} \sum_{j \in L_{U(i)}} \bar{u}_{ij} &= \sum_{j \in L_{U(i)}} P_{ij} = \sum_{j \in L_{U(i)}} (2^{\bar{u}_{ij}} - 1) \frac{N_0^j W_{ij}}{\delta_{ij}} \\ &\leq P_{imax}, \end{aligned}$$

The objective is to find linear matrix inequality over control variable signal $\bar{\mathbf{u}}_{ij}$ that satisfy above limitation constrain. According to the above nonlinear inequality we have:

$$\min_j \left\{ \frac{N_0^j W_{ij}}{\delta_{ij}} \right\} \sum_{j \in L_{U(i)}} (2^{\bar{u}_{ij}} - 1) \leq P_{imax}, \quad (48)$$

so

$$\sum_{j \in L_{U(i)}} 2^{\bar{u}_{ij}} \leq P_{imax} / S_i + l_i, S_i = \min_j \left\{ \frac{N_0^j W_{ij}}{\delta_{ij}} \right\} \quad (49)$$

So we are trying to use the linear estimation of $2^{\bar{u}_{ij}}$ around mean value of the resource (power) in each node ($\bar{P}_i = P_{imax} / l_i$), we will have:

$$\begin{aligned} \sum_{j \in L_{U(i)}} 2^{\bar{u}_{ij}} (1 + \ln 2) (\bar{\mathbf{u}}_{ij} - \hat{\mathbf{u}}_{ij}) &\leq \sum_{j \in L_{U(i)}} 2^{\bar{u}_{ij}} \\ &\leq P_{imax} / S_i + l_i, \end{aligned}$$

where $\hat{\mathbf{u}}_{ij} = \log_2 \left(1 + \frac{\delta_{ij} P_{imax} / l_i}{N_0^j W_{ij}} \right)$. So we

introduce the new linear constraint on control signal $\bar{\mathbf{u}}_{ij}$, as:

$$\begin{aligned} F_i \bar{\mathbf{u}}_i &\leq (P_{imax} / S_i + l_i) + \sum_{j \in L_{U(i)}} 2^{\bar{u}_{ij}} (1 + \ln 2) \hat{\mathbf{u}}_{ij} \\ F_i &= \text{vec}_j \{ 2^{\bar{u}_{ij}} (1 + \ln 2) \} \end{aligned}$$

References

- [1] L. Xiao, M. Johansson, and S. Boyd, "Simultaneous Routing and Resource Allocation Via Dual Decomposition", *IEEE Transactions on Communications*, Vol.52, No.7, pp.1136–1144, 2004.
- [2] N. A.Ribeiro and G.B.Giannakis, "Optimal Distributed Stochastic Routing Algorithms for Wireless Multihop Networks", *IEEE Transactions on Wireless Communications*, Vol.7, No.11, pp.4261– 4272, 2008.
- [3] J. H. Chang and L. Tassiulas, "Maximum Lifetime Routing in Wireless Sensor Networks", *IEEE/ACM Transaction on Network*, Vol.12, pp.609– 619, Aug 2004.
- [4] W. A.C.Valera and S.V.Rao, "Improving Protocol Robustness in ad hoc Networks Through Cooperative Packet Caching and Shortest Multipath Routing", *IEEE Transactions on Mobile Computing*, Vol.4, No.5, pp.443–457, 2005.
- [5] L. Chen and W. B. Heinzelman, "A Survey of Routing Protocols that Support QoS in Mobile ad hoc Networks", *IEEE Network*, Vol.21, pp.30– 38, Nov 2007.
- [6] C. Zhanga, J. Kurosea, Y. Liub, and D. Towsley, "An Optimal Distributed Algorithm for Joint Resource Allocation and Routing in Node-based Wireless Networks", *CMPSCI Technical Report*, pp.05-43, 2005.
- [7] R. Bhatia and M. Kodialam, "On Power Efficient Communication Over Multi-hop Wireless Networks: Joint Routing, Scheduling and Power Control", *INFOCOM*, Vol.2, No.1, pp.1457–1466, 2004.
- [8] T. Javidi and R. Cruz, "Distributed Link Scheduling, Power Control and Routing in Multi-hop Wireless Networks", *ACSSC*, pp.122–126, Oct 2006.
- [9] A. Sankar and Z. Liu, "Maximum Lifetime Routing in Wireless Ad-hoc Networks", *INFOCOM*, vol. 2, pp.1089–1097, March 2004.
- [11] A. Iftar and E. J. Davison, "Decentralized Control Strategies for Dynamic Routing", *Optimal Control Application and Methods*, Vol.23, pp.329– 355, Nov 2002.
- [12] Salman Baromand, M.Ali. Nekoei, K. Navaie, "Decentralized Covariance Control of Dynamic Routing in Traffic Networks: A Covariance Feedback Approach", *IET-Networks Journal*, Vol.1, Issue4, pp.289 – 301, 2012..
- [13] S. K. D. S. Ghosh and G. Ray, "Decentralized Stabilization of Uncertain Systems with Interconnection and Feedback Delays: An lmi Approach," *IEEE Transactions on Automatic Control*, Vol.54, No.4, 2009.
- [14] E. Fridman, "New Lyapunov-krasovskii Function for Stability of Linear Retarded and Neutral Type Systems", *System Control Lett*, Vol.43, pp.309–319, July 2001.
- [15] C. Q. Zhengrogn, Xiang and H. Weili, "Robust H₁ Control for Fuzzy Descriptor Systems with State Delay", *Journal of Systems Engineering and Electronics*, Vol.6, No.1, pp.98–101, 2005.
- [16] P. Baeir and S. Jung, "Advantages of CDMA and Spread Spectrum Techniques over FDMA and TDMA in Cellular Mobile Radio Applications", *IEEE Transaction on Vehicular Technology*, Vol.42, pp.357–364, Aug 1993.
- [17] A. J. van der Schaft, "L₂-gain Analysis of Nonlinear Systems and Nonlinear State Feedback H₁ Control", *IEEE Transactions on Automatic Control*, Vol.37, No.6, pp.770–784, 1992.
- [18] L. X. Y. Wand and D. Souza, "Robust Control of a Class of Uncertain Nonlinear Systems", *Syst. Control Lett*, Vol.19, pp.139–149, Aug 1992.