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Synchronization for Complex Dynamic Networks with State and Coupling Time-Delays

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Abstract

This paper is concerned with the problem of synchronization for complex dynamic networks with state and coupling time-delays. Therefore, larger class and more complicated complex dynamic networks can be considered for the synchronization problem. Based on the Lyapunov-Krasovskii functional, a delay-independent criterion is obtained and formulated in the form of linear matrix inequalities (LMIs) to ascertain the synchronization between each nodes of the complex dynamic network. The effectiveness of the proposed method is illustrated using a numerical simulation.

Keywords: Complex Dynamic Network; Synchronization; Lyapunov–Krasovskii; Time-Delayed Systems

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1. Introduction

Many systems in the real-world can be modeled by networks, such as the electrical power grids, neural networks, social network, communication networks, the Internet, and the World Wide Web [1]. Complex networks are made up of interconnected nodes interacting with the others via a topology defined on the network edges [2-5]. These nodes are representing the individuals in the network with different meaning in different situations [6]. Each node of the network can be a nonlinear dynamical system and create a complex dynamic network (CDN), which have been widely applied to model many complex systems. In the past few decades, the study of CDNs has received increasing attention from researchers in various disciplines, such as physics, mathematics, engineering, biology, and sociology [7-11].

Synchronization among all network's dynamical nodes is one of the most typical collective behavior and basic motions in nature and is one of the most interesting and significant phenomena in CDNs [12-18]. In general, time delays occur commonly in networks because of the network traffic congestion as well as the finite speed of signal transmission over the links. Hence, the synchronization study of CDNs with coupling time delays is quite important [19-22]. Exponential synchronization in CDNs with time-varying delay and hybrid coupling is investigated in [21]. Guaranteed cost synchronization of CDNs is introduced in [23-25]. To the best of the author's knowledge, complex dynamic networks with time-delay in the states of dynamical nodes have been rarely studied. In [26], synchronization criterion for Lur'e type complex dynamical networks are considered with time-delay in the states of nodes, but the coupling delay between nodes is not considered.

In this paper, synchronization criteria for complex dynamic networks with state and coupling time-delays are presented. Therefore, larger class and more complicated CDNs can be considered for the synchronization problem. Based on the Lyapunov-Krasovskii functional approach, a delay-independent criterion is obtained and formulated in the form of LMIs. The effectiveness of the proposed method is illustrated using some numerical simulations.

The organization of this paper is as follows. In Section 2, the problem formulation for the complex dynamic network structure with state and coupling time-delays are presented. In Section 3, based on the Lyapunov–Krasovskii functional and LMI, a criterion is given to ascertain the synchronization between the nodes of complex dynamic network. Section 4 provides simulation results. Finally, section 5 concludes the paper.

Notations. Throughout this paper, denotes the n-dimensional Euclidean space and is the set of real $n\times m$ matrices. $\mathbf{P}>0$ means that $\mathbf{P}$ is a real positive definite and symmetric matrix. $\mathbf{I}$ is the identity matrix with appropriate dimensions and $\mathrm{diag}\{\mathbf{W}_1,\dots,\mathbf{W}_m\}$ refers to a real matrix with diagonal elements $\mathbf{W}_1,\dots,\mathbf{W}_m$. $\mathbf{A}^T$ denotes the transpose of the real matrix $\mathbf{A}$. Symmetric terms in a symmetric matrix are denoted by $*$ and the sign $\otimes$ is stand for the Kronecker product.

2. Problem Statement and preliminaries

Consider a complex dynamic network with N delayed identical nodes with coupling delay:

$$\dot{\mathbf{x}}_i(t)=\mathbf{A}\mathbf{x}_i(t)+\mathbf{A_d}\mathbf{x}_i(t-\tau)+\mathbf{Bf}(\mathbf{M}\mathbf{x}_i(t))+\mathbf{Cf}(\mathbf{D}\mathbf{x}_i(t-\tau)) \quad (1$$
$$+\sum_{j=1}^{N}G_{ij}^{(1)}\mathbf{\Gamma}_1\mathbf{x}_i(t)+\sum_{j=1}^{N}G_{ij}^{(2)}\mathbf{\Gamma}_2\mathbf{x}_i(t-\tau_c), i=1,2,\dots,N$$

where

$\mathbf{x}_i(t)=\begin{bmatrix}x_{i1}(t) & x_{i2}(t) & \cdots & x_{in}(t)\end{bmatrix}^T\in\mathbb{R}^n$ denote the state vector of node i, $\mathbf{f}:\mathbb{R}^n\to\mathbb{R}^n$ is a nonlinear vector-valued function, $\mathbf{A},\mathbf{A_d},\mathbf{B},\mathbf{M},\mathbf{C},\mathbf{D}\in\mathbb{R}^{n\times n}$ are constant matrices, $\tau>0$ denotes the state delay and $\tau_c>0$ is the coupling delay. $\mathbf{G}^{(q)}=(G_{ij}^{(q)})_{N\times N},(q=1,2)$ denotes the coupling connections and $\mathbf{\Gamma}_1,\mathbf{\Gamma}_2\in\mathbb{R}^{n\times n}$ represent the inner coupling matrices.

Remark 1. To the best of the author knowledge, this model is not considered yet. In this model, the state delay (τ) could be different to coupling delay (τ_c). Hence, more general complex dynamic networks could be modeled in this way.

Assumption 1. The coupling connection matrices should satisfy

$$\begin{cases}G_{ij}^{(q)}=G_{ji}^{(q)}\geq 0, & i\neq j, q=1,2,\\ G_{ii}^{(q)}=-\sum_{j=1,j\neq i}^{N}G_{ij}^{(q)}\geq 0, & i,j=1,\dots,N, q=1,2.\end{cases}$$

Throughout this paper, we make the following assumption on $\mathbf{f}(.)$.

Assumption 2. For any $x_1,x_2\in\mathbb{R}$ there are some constants, σ_r^-,σ_r^+, which the nonlinear function satisfies

$$\sigma_r^-\leq\frac{f_r(x_1)-f_r(x_2)}{x_1-x_2}\leq\sigma_r^+,\quad r=1,2,\dots,n.$$

For notation simplicity, let

$$\mathbf{x}(t)=\begin{bmatrix}\mathbf{x}_1^T(t) & \mathbf{x}_2^T(t) & \cdots & \mathbf{x}_N^T(t)\end{bmatrix}^T$$

With the help of the matrix Kronecker product, the network **Error! Reference source not found.** can be written as the following form:

(1)

$$\dot{\mathbf{x}}(t)=(\mathbf{I}_N\otimes\mathbf{A})\mathbf{x}(t)+(\mathbf{I}_N\otimes\mathbf{A_d})\mathbf{x}(t-\tau)+(\mathbf{I}_N\otimes\mathbf{B})\times \quad (2)$$
$$\mathbf{F}((\mathbf{I}_N\otimes\mathbf{M})\mathbf{x}(t))+(\mathbf{I}_N\otimes\mathbf{C})\mathbf{F}((\mathbf{I}_N\otimes\mathbf{D})\mathbf{x}(t-\tau))$$
$$+\left(\mathbf{G}^{(1)}\otimes\mathbf{\Gamma}_1\right)\mathbf{x}(t)+\left(\mathbf{G}^{(2)}\otimes\mathbf{\Gamma}_2\right)\mathbf{x}(t-\tau_c),$$

The following definition and lemmas will be needed in the derivations of our main results.

Definition1. The system **Error! Reference source not found.** is said to be globally synchronized for any initial conditions $\Pi_{i0}(s),(i=1,2,\dots,N)$, if the following holds:

$$\lim_{t\to\infty}\|\mathbf{x}_i(t)-\mathbf{x}_j(t)\|=0,\qquad\forall i,j=1,2,\dots,N,$$

where $\|.\|$ denotes Euclidean norm.

Lemma 1. ([8]). Let $\alpha\in\mathbb{R}$ and $\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}$ be matrices with appropriate dimensions. The following properties can be proved

- $(\alpha\mathbf{A})\otimes\mathbf{B}=\mathbf{A}\otimes(\alpha\mathbf{B})$
- $(\mathbf{A}\otimes\mathbf{B})^T=\mathbf{A}^T\otimes\mathbf{B}^T$
- $(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D})=(\mathbf{AC})\otimes(\mathbf{BD})$
- $\mathbf{A}\otimes\mathbf{B}\otimes\mathbf{C}=(\mathbf{A}\otimes\mathbf{B})\otimes\mathbf{C}=\mathbf{A}\otimes(\mathbf{B}\otimes\mathbf{C})$

Lemma 2. ((Jensen Inequality), [19]). Assume that the vector function $\boldsymbol{\omega}:[0,r]\to\mathbb{R}^n$ is well defined for the following integrations. For any symmetric matrix $\mathbf{R}\in\mathbb{R}^{n\times n}$ and scalar $r>0$, one has

$$r\int_0^r\boldsymbol{\omega}^T(s)\mathbf{R}\boldsymbol{\omega}(s)ds\geq\left(\int_0^r\boldsymbol{\omega}(s)ds\right)^T\mathbf{R}\left(\int_0^r\boldsymbol{\omega}(s)ds\right).$$

Lemma 3. According to [27] and Assumption 2, for any diagonal matrices $\mathbf{J}>0,\mathbf{L}>0$, and constant matrices $\mathbf{M}$ and $\mathbf{D}$ with appropriate dimensions, it follows that

$$\boldsymbol{\theta}^T(t)\begin{bmatrix}-\mathbf{M}^T\mathbf{J}\mathbf{\Delta}_1\mathbf{M} & \mathbf{M}^T\mathbf{J}\mathbf{\Delta}_2\\ * & -\mathbf{J}\end{bmatrix}\boldsymbol{\theta}(t) \quad (3$$
$$+\boldsymbol{\theta}^T(t-\tau)\begin{bmatrix}-\mathbf{M}^T\mathbf{L}\mathbf{\Delta}_1\mathbf{M} & \mathbf{M}^T\mathbf{L}\mathbf{\Delta}_2\\ * & -\mathbf{L}\end{bmatrix}\boldsymbol{\theta}(t-\tau)\geq 0,$$

where

$$\boldsymbol{\theta}(t)=\begin{bmatrix}\mathbf{x}_i(t)-\mathbf{x}_j(t)\\ \mathbf{f}(\mathbf{Mx}_i(t))-\mathbf{f}(\mathbf{Mx}_j(t))\end{bmatrix},$$

$$\boldsymbol{\Delta}_1=diag\left[\sigma_1^+\sigma_1^-,\ldots,\sigma_n^+\sigma_n^-\right],$$

$$\boldsymbol{\Delta}_2=\mathrm{diag}\left[\frac{\sigma_1^++\sigma_1^-}{2},\ldots,\frac{\sigma_n^++\sigma_n^-}{2}\right].$$

Lemma 4. ([28]). Let $\mathbf{e}=[1,1,\ldots,1]^T$, $\mathbf{E}_N=\mathbf{ee}^T$, and $\mathbf{U}=N\mathbf{I}_N-\mathbf{E}_N$, $\mathbf{P}\in\mathbb{R}^{n\times n}$, $\mathbf{x}=\left[\mathbf{x}_1^T,\ldots,\mathbf{x}_N^T\right]^T$, and $\mathbf{y}=\left[\mathbf{y}_1^T,\ldots,\mathbf{y}_N^T\right]^T$ with $\mathbf{x}_k,\mathbf{y}_k\in\mathbb{R}^n$, $(k=1,2,\ldots,\mathrm{N})$, then

$$\mathbf{x}^T(\mathbf{U}\otimes\mathbf{P})\mathbf{y}=\sum_{1\le i<j\le N}^{N}(\mathbf{x}_i-\mathbf{x}_j)^T\mathbf{P}(\mathbf{y}_i-\mathbf{y}_j).$$

Lemma 5. ([28]). Let H, S be $n\times n$ any real matrix, $\mathbf{e}=[1,1,\ldots,1]^T$, $\mathbf{E}_N=\mathbf{ee}^T$, $\mathbf{U}=N\mathbf{I}_N-\mathbf{E}_N$, $\mathbf{x}=\left[\mathbf{x}_1^T,\ldots,\mathbf{x}_N^T\right]^T$, and $\mathbf{y}=\left[\mathbf{y}_1^T,\ldots,\mathbf{y}_N^T\right]^T$ with $\mathbf{x}_k,\mathbf{y}_k\in\mathbb{R}^n$, $(k=1,2,\ldots,\mathrm{N})$, and $f(.)$ and $F(.)$ are functions and defined in (1). Then, for any vectors $\mathbf{x}$ and $\mathbf{y}$ with appropriate dimensions, the following matrix inequality holds:

$$\mathbf{x}^T(\mathbf{U}\otimes\mathbf{P})\mathbf{F}((\mathrm{I}_N\otimes\mathbf{S})\mathbf{y})=\sum_{1\le i<j\le N}^{N}(\mathbf{x}_i-\mathbf{x}_j)^T\mathbf{H}\left(\mathbf{f}(\mathbf{Sy}_i)-\mathbf{f}(\mathbf{Sy}_j)\right)$$

3. Main Result

In this chapter, a sufficient condition based on the Lyapunov-Krasovskii functional method will be presented for the synchronization between the nodes of complex dynamic network (1).

Theorem 1. The system (1) is globally synchronized if there exist positive definite matrices $\mathbf{P}>0$, $\mathbf{Q}>0$, $\mathbf{R}>0$, and positive diagonal matrices $\mathbf{J}_1>0$, $\mathbf{J}_2>0$, $\mathbf{L}_1>0$, $\mathbf{L}_2>0$, such that the following LMIs hold for all $1\le i<j\le N$:

$$\boldsymbol{\Xi}_{ij}=\begin{bmatrix}\Pi_{11} & \mathbf{PA}_d & \Pi_{13} & \Pi_{14} & 0 & \Pi_{16} & \mathbf{PC}\\ * & \Pi_{22} & 0 & 0 & \Pi_{25} & 0 & \Pi_{27}\\ * & * & -\mathbf{R} & 0 & 0 & 0 & 0\\ * & * & * & \Pi_{44} & 0 & \mathbf{Q}_{23} & 0\\ * & * & * & * & \Pi_{55} & 0 & -\mathbf{Q}_{23}\\ * & * & * & * & * & \Pi_{66} & 0\\ * & * & * & * & * & * & \Pi_{77}\end{bmatrix}<0 \quad (4)$$

$$\mathbf{Q}=\begin{bmatrix}\mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{Q}_{13}\\ * & \mathbf{Q}_{22} & \mathbf{Q}_{23}\\ * & * & \mathbf{Q}_{33}\end{bmatrix}>0,$$

where

$$\Pi_{11}=\mathbf{PA}+\mathbf{A}^T\mathbf{P}+\mathbf{Q}_{11}+\mathbf{R}-\mathbf{M}^T\mathbf{J}_1\boldsymbol{\Delta}_1\mathbf{M}-\mathbf{D}^T\mathbf{J}_2\boldsymbol{\Delta}_1\mathbf{D}-NG_{ij}^{(1)}\mathbf{P}\boldsymbol{\Gamma}_1-NG_{ij}^{(1)}\boldsymbol{\Gamma}_1^T\mathbf{P},$$

$\Pi_{13}=-NG_{ij}^{(2)}\mathbf{P}\boldsymbol{\Gamma}_2$, $\Pi_{14}=\mathbf{PB}+\mathbf{Q}_{12}+\mathbf{M}^T\mathbf{J}_1\boldsymbol{\Delta}_2$,

$\Pi_{16}=\mathbf{Q}_{13}+\mathbf{D}^T\mathbf{J}_2\boldsymbol{\Delta}_2$,

$\Pi_{22}=-\mathbf{Q}_{11}-\mathbf{M}^T\mathbf{L}_1\boldsymbol{\Delta}_1\mathbf{M}-\mathbf{D}^T\mathbf{L}_2\boldsymbol{\Delta}_1\mathbf{D}$,

$\Pi_{25}=-\mathbf{Q}_{12}+\mathbf{M}^T\mathbf{L}_1\boldsymbol{\Delta}_2$, $\Pi_{27}=-\mathbf{Q}_{13}+\mathbf{D}^T\mathbf{L}_2\boldsymbol{\Delta}_2$,

$\Pi_{44}=\mathbf{Q}_{22}-\mathbf{J}_1$, $\Pi_{55}=-\mathbf{Q}_{22}-\mathbf{L}_1$, $\Pi_{66}=\mathbf{Q}_{33}-\mathbf{J}_2$,

$\Pi_{77}=-\mathbf{Q}_{33}-\mathbf{L}_2$.

Proof. Consider the following Lyapunov-Krasovskii functional:

$$V(t)=V_1(t)+V_2(t)+V_3(t) \quad (1)$$

where

$$V_1(t)=\mathbf{x}^T(t)(\mathbf{U}\otimes\mathbf{P})\mathbf{x}(t),$$

$$V_2(t)=\int_{t-\tau}^{t}\begin{bmatrix}\mathbf{x}(s)\\ \mathbf{F}(\mathbf{Mx}(s))\\ \mathbf{F}(\mathbf{Dx}(s))\end{bmatrix}^T\begin{bmatrix}\mathbf{U}\otimes\mathbf{Q}_{11} & \mathbf{U}\otimes\mathbf{Q}_{12} & \mathbf{U}\otimes\mathbf{Q}_{13}\\ * & \mathbf{U}\otimes\mathbf{Q}_{22} & \mathbf{U}\otimes\mathbf{Q}_{23}\\ * & * & \mathbf{U}\otimes\mathbf{Q}_{33}\end{bmatrix}\begin{bmatrix}\mathbf{x}(s)\\ \mathbf{F}(\mathbf{Mx}(s))\\ \mathbf{F}(\mathbf{Dx}(s))\end{bmatrix}ds,$$

$$V_3(t)=\int_{t-\tau_c}^{t}\mathbf{x}^T(s)(\mathbf{U}\otimes\mathbf{R})\mathbf{x}(s)ds,$$

where $\mathbf{U}$ is defined in Lemma 4.

Taking the derivative of $V_1(t)$ with respect to t yields:

$$\dot{V}_1(t)=2\mathbf{x}^T(t)(\mathbf{U}\otimes\mathbf{P})\dot{\mathbf{x}}(t)=2\mathbf{x}^T(t)(\mathbf{U}\otimes\mathbf{P})\Big[(\mathbf{I}_N\otimes\mathbf{A})\mathbf{x}(t)+(\mathbf{I}_N\otimes\mathbf{A}_d)\mathbf{x}(t-\tau)+(\mathbf{I}_N\otimes\mathbf{B})\mathbf{F}((\mathbf{I}_N\otimes\mathbf{M})\mathbf{x}(t))+(\mathbf{I}_N\otimes\mathbf{C})\mathbf{F}((\mathbf{I}_N\otimes\mathbf{D})\mathbf{x}(t-\tau))+(\mathbf{G}^{(1)}\otimes\boldsymbol{\Gamma}_1)\mathbf{x}(t)+(\mathbf{G}^{(2)}\otimes\boldsymbol{\Gamma}_2)\mathbf{x}(t-\tau_c)\Big] \quad (\text{[illegible]}$$

According to Lemma 4, **Error! Reference source not found.** can be written as the following:

$$\dot{V}_1(t)=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\Big[2(\mathbf{x}_i(t)-\mathbf{x}_j(t))^T\Big[\mathbf{PA}(\mathbf{x}_i(t)-\mathbf{x}_j(t))+\mathbf{PA}_d(\mathbf{x}_i(t-\tau)-\mathbf{x}_j(t-\tau))+\mathbf{PB}(\mathbf{f}(\mathbf{Mx}_i(t))-\mathbf{f}(\mathbf{Mx}_j(t)))+\mathbf{PC}(\mathbf{f}(\mathbf{Dx}_i(t-\tau))-\mathbf{f}(\mathbf{Dx}_j(t-\tau)))-NG_{ij}^{(1)}\mathbf{P}\boldsymbol{\Gamma}_1(\mathbf{x}_i(t)-\mathbf{x}_j(t))-NG_{ij}^{(2)}\mathbf{P}\boldsymbol{\Gamma}_2(\mathbf{x}_i(t-\tau_c)-\mathbf{x}_j(t-\tau_c))\Big]\Big]. \quad (\text{[illegible]}$$

The second term of **Error! Reference source not found.** becomes

$$\dot{V}_2(t)=\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{F}(\mathbf{Mx}(t))\\ \mathbf{F}(\mathbf{Dx}(t))\end{bmatrix}^T\begin{bmatrix}\mathbf{U}\otimes\mathbf{Q}_{11} & \mathbf{U}\otimes\mathbf{Q}_{12} & \mathbf{U}\otimes\mathbf{Q}_{13}\\ * & \mathbf{U}\otimes\mathbf{Q}_{22} & \mathbf{U}\otimes\mathbf{Q}_{23}\\ * & * & \mathbf{U}\otimes\mathbf{Q}_{33}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{F}(\mathbf{Mx}(t))\\ \mathbf{F}(\mathbf{Dx}(t))\end{bmatrix}$$
$$-\begin{bmatrix}\mathbf{x}(t-\tau)\\ \mathbf{F}(\mathbf{Mx}(t-\tau))\\ \mathbf{F}(\mathbf{Dx}(t-\tau))\end{bmatrix}^T\begin{bmatrix}\mathbf{U}\otimes\mathbf{Q}_{11} & \mathbf{U}\otimes\mathbf{Q}_{12} & \mathbf{U}\otimes\mathbf{Q}_{13}\\ * & \mathbf{U}\otimes\mathbf{Q}_{22} & \mathbf{U}\otimes\mathbf{Q}_{23}\\ * & * & \mathbf{U}\otimes\mathbf{Q}_{33}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t-\tau)\\ \mathbf{F}(\mathbf{Mx}(t-\tau))\\ \mathbf{F}(\mathbf{Dx}(t-\tau))\end{bmatrix}$$

According to Lemma (4) and (5), **Error! Reference source not found.** can be written as the following:

$$\begin{aligned}\dot{V}_2(t)=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\Big[&\left(\mathbf{x}_i(t)-\mathbf{x}_j(t)\right)^T\mathbf{Q}_{11}\left(\mathbf{x}_i(t)-\mathbf{x}_j(t)\right)\\
&+2\left(\mathbf{x}_i(t)-\mathbf{x}_j(t)\right)^T\mathbf{Q}_{12}\left(\mathbf{f}(\mathbf{Mx}_i(t))-\mathbf{f}(\mathbf{Mx}_j(t))\right)\\
&+2\left(\mathbf{x}_i(t)-\mathbf{x}_j(t)\right)^T\mathbf{Q}_{13}\left(\mathbf{f}(\mathbf{Dx}_i(t))-\mathbf{f}(\mathbf{Dx}_j(t))\right)\\
&+\left(\mathbf{f}(\mathbf{Mx}_i(t))-\mathbf{f}(\mathbf{Mx}_j(t))\right)^T\mathbf{Q}_{22}\left(\mathbf{f}(\mathbf{Mx}_i(t))-\mathbf{f}(\mathbf{Mx}_j(t))\right)\\
&+2\left(\mathbf{f}(\mathbf{Mx}_i(t))-\mathbf{f}(\mathbf{Mx}_j(t))\right)^T\mathbf{Q}_{23}\left(\mathbf{f}(\mathbf{Dx}_i(t))-\mathbf{f}(\mathbf{Dx}_j(t))\right)\\
&+\left(\mathbf{f}(\mathbf{Dx}_i(t))-\mathbf{f}(\mathbf{Dx}_j(t))\right)^T\mathbf{Q}_{33}\left(\mathbf{f}(\mathbf{Dx}_i(t))-\mathbf{f}(\mathbf{Dx}_j(t))\right)\\
&-\left(\mathbf{x}_i(t-\tau)-\mathbf{x}_j(t-\tau)\right)^T\mathbf{Q}_{11}\left(\mathbf{x}_i(t-\tau)-\mathbf{x}_j(t-\tau)\right)\\
&-2\left(\mathbf{x}_i(t-\tau)-\mathbf{x}_j(t-\tau)\right)^T\mathbf{Q}_{12}\left(\mathbf{f}(\mathbf{Mx}_i(t-\tau))-\mathbf{f}(\mathbf{Mx}_j(t-\tau))\right)\\
&-2\left(\mathbf{x}_i(t-\tau)-\mathbf{x}_j(t-\tau)\right)^T\mathbf{Q}_{13}\left(\mathbf{f}(\mathbf{Dx}_i(t-\tau))-\mathbf{f}(\mathbf{Dx}_j(t-\tau))\right)\\
&-\left(\mathbf{f}(\mathbf{Mx}_i(t-\tau))-\mathbf{f}(\mathbf{Mx}_j(t-\tau))\right)^T\mathbf{Q}_{22}\left(\mathbf{f}(\mathbf{Mx}_i(t-\tau))-\mathbf{f}(\mathbf{Mx}_j(t-\tau\right.\\
&-2\left(\mathbf{f}(\mathbf{Mx}_i(t-\tau))-\mathbf{f}(\mathbf{Mx}_j(t-\tau))\right)^T\mathbf{Q}_{23}\left(\mathbf{f}(\mathbf{Dx}_i(t-\tau))-\mathbf{f}(\mathbf{Dx}_j(t-\tau\right.\\
&-\left(\mathbf{f}(\mathbf{Dx}_i(t-\tau))-\mathbf{f}(\mathbf{Dx}_j(t-\tau))\right)^T\mathbf{Q}_{33}\left(\mathbf{f}(\mathbf{Dx}_i(t-\tau))-\mathbf{f}(\mathbf{Dx}_j(t-\tau))\right)\end{aligned}$$

The third term of **Error! Reference source not found.** becomes

$$\begin{aligned}\dot{V}_3(t)&=\mathbf{x}^T(t)(\mathbf{U}\otimes\mathbf{R})\mathbf{x}(t)-\mathbf{x}^T(t-\tau_c)(\mathbf{U}\otimes\mathbf{R})\mathbf{x}(t-\tau_c)\\
&=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\Big[\left(\mathbf{x}_i(t)-\mathbf{x}_j(t)\right)^T\mathbf{R}\left(\mathbf{x}_i(t)-\mathbf{x}_j(t)\right)\\
&\quad-\left(\mathbf{x}_i(t-\tau_c)-\mathbf{x}_j(t-\tau_c)\right)^T\mathbf{R}\left(\mathbf{x}_i(t-\tau_c)-\mathbf{x}_j(t-\tau_c)\right)\Big]\end{aligned}\tag{6}$$

According to Lemma 3 and Assumption 2, for any positive diagonal matrices $\mathbf{J}_1,\mathbf{J}_2,\mathbf{L}_1,\mathbf{L}_2$, one has

$$\boldsymbol{\theta}^T(t)\begin{bmatrix}-\mathbf{M}^T\mathbf{J}_1\boldsymbol{\Delta}_1\mathbf{M} & \mathbf{M}^T\mathbf{J}_1\boldsymbol{\Delta}_2\\ * & -\mathbf{J}_1\end{bmatrix}\boldsymbol{\theta}(t)+\boldsymbol{\theta}^T(t-\tau)\begin{bmatrix}-\mathbf{M}^T\mathbf{L}_1\boldsymbol{\Delta}_1\mathbf{M} & \mathbf{M}^T\mathbf{L}_1\boldsymbol{\Delta}_2\\ * & -\mathbf{L}_1\end{bmatrix}\boldsymbol{\theta}(t-\tau)\geq 0\tag{7}$$

$$\boldsymbol{\beta}^T(t)\begin{bmatrix}-\mathbf{D}^T\mathbf{J}_2\boldsymbol{\Delta}_1\mathbf{D} & \mathbf{D}^T\mathbf{J}_2\boldsymbol{\Delta}_2\\ * & -\mathbf{J}_2\end{bmatrix}\boldsymbol{\beta}(t)+\boldsymbol{\beta}^T(t-\tau)\begin{bmatrix}-\mathbf{D}^T\mathbf{L}_2\boldsymbol{\Delta}_1\mathbf{D} & \mathbf{D}^T\mathbf{L}_2\boldsymbol{\Delta}_2\\ * & -\mathbf{L}_2\end{bmatrix}\boldsymbol{\beta}(t-\tau)\geq 0\tag{8}$$

where

$$\boldsymbol{\theta}(t)=\begin{bmatrix}\mathbf{x}_i(t)-\mathbf{x}_j(t)\\ \mathbf{f}(\mathbf{Mx}_i(t))-\mathbf{f}(\mathbf{Mx}_j(t))\end{bmatrix},$$
$$\boldsymbol{\beta}(t)=\begin{bmatrix}\mathbf{x}_i(t)-\mathbf{x}_j(t)\\ \mathbf{f}(\mathbf{Dx}_i(t))-\mathbf{f}(\mathbf{Dx}_j(t))\end{bmatrix}.$$

Considering **Error! Reference source not found.**-- **Error! Reference source not found.**, it is straightforward to show that

$$\dot{V}(t)\leq\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left[\boldsymbol{\xi}_{ij}^T(t)\,\boldsymbol{\Xi}_{ij}\boldsymbol{\xi}_{ij}(t)\right]\tag{9}$$

where $\boldsymbol{\Xi}_{ij}$ is defined in **Error! Reference source not found.** and

$$\begin{aligned}\boldsymbol{\xi}_{ij}(t)=\Big[&\left(\mathbf{x}_i(t)-\mathbf{x}_j(t)\right)^T,\left(\mathbf{x}_i(t-\tau)-\mathbf{x}_j(t-\tau)\right)^T\\
&\left(\mathbf{x}_i(t-\tau_c)-\mathbf{x}_j(t-\tau_c)\right)^T,\left(\mathbf{f}(\mathbf{Mx}_i(t))-\mathbf{f}(\mathbf{Mx}_j(t))\right)^T\\
&\left(\mathbf{f}(\mathbf{Mx}_i(t-\tau))-\mathbf{f}(\mathbf{Mx}_j(t-\tau))\right)^T,\left(\mathbf{f}(\mathbf{Dx}_i(t))-\mathbf{f}(\mathbf{Dx}_j(t))\right)^T\\
&\left(\mathbf{f}(\mathbf{Dx}_i(t-\tau))-\mathbf{f}(\mathbf{Dx}_j(t-\tau))\right)^T\Big]^T.\end{aligned}$$

If $\boldsymbol{\Xi}_{ij}<0$ for $\forall 1\leq i<j\leq N$, then $\dot{V}(t)<0$. From Definition 1, this implies that the system (1) has a global synchronization. This completes the proof.

Remark 2. Theorem 1 provides delay-independent criterion. If Theorem 1 satisfied for a system, then the system has global synchronization for any state delay (τ) and coupling delay (τ_c). Obviously this criterion is a conservative condition.

4. Illustrative Example

Example. Consider the system (1) with the following parameters [28]:

$$\mathbf{A}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix},\quad \mathbf{A}_d=\begin{bmatrix}-1.2 & 0.8\\ -0.2 & -0.2\end{bmatrix},\quad \mathbf{B}=\mathbf{C}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$
$$\mathbf{M}=\begin{bmatrix}3.8 & 2\\ 0.1 & 1.8\end{bmatrix},\quad \mathbf{D}=\begin{bmatrix}-3.5 & 1\\ 0.1 & -1.5\end{bmatrix},\quad \boldsymbol{\Gamma}_1=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix},$$
$$\boldsymbol{\Gamma}_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ \boldsymbol{\Delta}_1=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\ \boldsymbol{\Delta}_2=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix},$$

and

$$\mathbf{G}^{(1)}=\mathbf{G}^{(2)}=\begin{bmatrix}-5 & 1 & 1 & 1 & 1 & 1\\ 1 & -5 & 1 & 1 & 1 & 1\\ 1 & 1 & -5 & 1 & 1 & 1\\ 1 & 1 & 1 & -5 & 1 & 1\\ 1 & 1 & 1 & 1 & -5 & 1\\ 1 & 1 & 1 & 1 & 1 & -5\end{bmatrix}.$$

By applying Theorem 1 into this example, it is shown that this system can achieve global synchronization with any admissible time delay. For $f(x)=0.25(|x+1|-|x-1|)$, $\tau=0.6$, and $\tau_c=0.7$, the states trajectories and the synchronization errors are shown in figures 1 and 2, where $\mathbf{e}_j(t)=\left(\mathbf{x}_{ij}(t)-\mathbf{x}_{1j}(t)\right)$, $i=2,\ldots,6;\ j=1,2$.

5. Conclusion

This paper is considered the problem of synchronization for complex dynamic networks with state and coupling time-delays. Based on the Lyapunov-Krasovskii functional, a delay-independent criterion was obtained and formulated in the form of linear matrix inequalities (LMIs) to ascertain the synchronization between each nodes of the complex dynamic network. The effectiveness of the proposed method was illustrated using a numerical simulation.

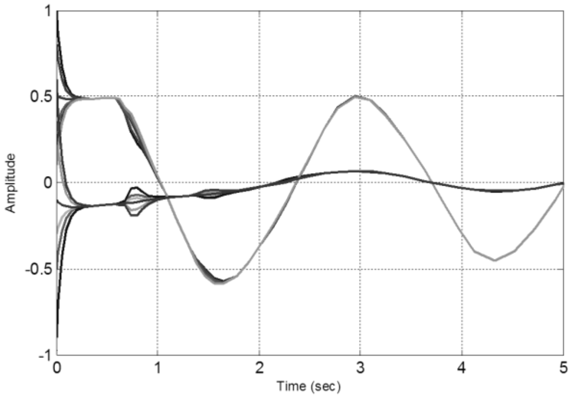

Fig. 1. State trajectories: $\mathbf{x}_i(t); i = 1,2$.

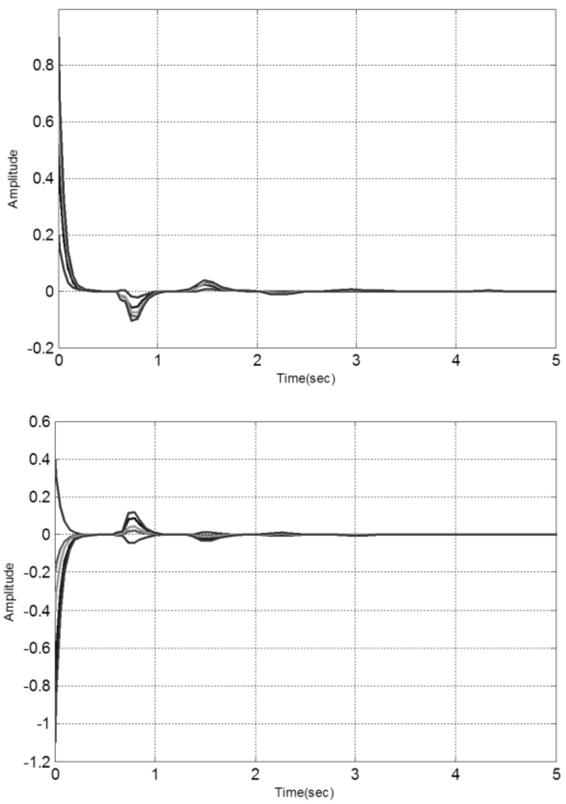

Fig. 2. Synchronization errors for the network: $e_1(t), e_2(t)$

References

[1] R. Lu, W. Yu, J. Lu, and A. Xue, "Synchronization on complex networks of networks," IEEE Transactions on Neural Networks and Learning Systems, vol. 25, pp. 2110-2118 , 2014.

[2] L. Cui, S. Kumara, and R. Albert, "Complex networks: An engineering view," IEEE Circuits and Systems Magazine, vol. 10, pp. 10-25, 2010.

[3] S. Manaffam and A. Seyedi, "Synchronization Probability in Large Complex Networks," IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 60, pp. 697-701, 2013.

[4] M. E. Newman, "The structure and function of complex networks," SIAM review, vol. 45, pp. 167-256, 2003.

[5] W. Yu, P. DeLellis, G. Chen, M. D. Bernardo, and J. Kurths, "Distributed adaptive control of synchronization in complex networks," IEEE Transactions on Automatic Control, vol. 57, pp. 2153-2158, 2012.

[6] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: Structure and dynamics," Physics reports, vol. 424, pp. 175-308, 2006.

[7] C. Cai, Z. Wang, J. Xu, X. Liu, and F. E. Alsaadi, "An Integrated Approach to Global Synchronization and State Estimation for Nonlinear Singularly Perturbed Complex Networks," IEEE Transactions on Cybernetics, vol. 45, pp. 1597-1609, 2014.

[8] H. Li, Z. Ning, Y. Yin, and Y. Tang, "Synchronization and state estimation for singular complex dynamical networks with time-varying delays," Communications in Nonlinear Science and Numerical Simulation, vol. 18, pp. 194-208, 2013.

[9] J. Lü and G. Chen, "A time-varying complex dynamical network model and its controlled synchronization criteria," IEEE Transactions on Automatic Control, vol. 50, pp. 841-846, 2005.

[10] N. Mahdavi, M. B. Menhaj, J. Kurths, and J. Lu, "Fuzzy complex dynamical networks and its synchronization," IEEE Transactions on Cybernetics, vol. 43, pp. 648-659, 2013.

[11] B. Shen, Z. Wang, and X. Liu, "Bounded H∞ synchronization and state estimation for discrete time-varying stochastic complex for discrete time-varying stochastic complex networks over a finite horizon," IEEE Transactions on Neural Networks, vol. 22, pp. 145-157, 2011.

[12] C. Li, W. Sun, and J. Kurths, "Synchronization of complex dynamical networks with time delays," Physica A: Statistical Mechanics and its Applications, vol. 361, pp. 24-34, 2006.

[13] H. Li and D. Yue, "Synchronization stability of general complex dynamical networks with time-varying delays: A piecewise analysis method," Journal of computational and applied mathematics, vol. 232, pp. 149-158, 2009.

[14] T. Liu, J. Zhao, and D. J. Hill, "Exponential synchronization of complex delayed dynamical networks with switching topology," IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 57, pp. 2967-2980, 2010.

[15] B. Wang and Z.-H. Guan, "Chaos synchronization in general complex dynamical networks with coupling delays," Nonlinear Analysis: Real World Applications, vol. 11, pp. 1925-1932, 2010.

[16] D. Yue and H. Li, "Synchronization stability of continuous/discrete complex dynamical networks with interval time-varying delays," Neurocomputing, vol. 73, pp. 809-819, 2010.

[17] Y. Zhang, D.-W. Gu, and S. Xu, "Global exponential adaptive synchronization of complex dynamical networks with neutral-type neural network nodes and stochastic disturbances," IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 60, pp. 2709-2718, 2013.

[18] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, "Synchronization in complex networks," Physics Reports, vol. 469, pp. 93-153, 2008.

[19] M. Fang, "Synchronization for complex dynamical networks with time delay and discrete-time information," Applied Mathematics and Computation, vol. 258, pp. 1-11, 2015.

[20] Y. Sun, W. Li, and J. Ruan, "Generalized outer synchronization between complex dynamical networks with time delay and noise perturbation," Communications in Nonlinear Science and Numerical Simulation, vol. 18, pp. 989-998, 2013.

[21] J. Wang, H. Zhang, Z. Wang, and B. Wang, "Local exponential synchronization in complex dynamical networks with time-varying delay and hybrid coupling," Applied Mathematics and Computation, vol. 225, pp. 16-32, 2013.

[22] L. Zhang, Y. Wang, Y. Huang, and X. Chen, "Delay-dependent synchronization for non-diffusively coupled time-varying complex dynamical networks," Applied Mathematics and Computation, vol. 259, pp. 510-522, 2015.

[23] T. H. Lee, D. Ji, J. H. Park, and H. Y. Jung, "Decentralized guaranteed cost dynamic control for synchronization of a complex dynamical network with randomly switching topology," Applied Mathematics and Computation, vol. 219, pp. 996-1010, 2012.

[24] T. H. Lee, J. H. Park, D. Ji, O. Kwon, and S.-M. Lee, "Guaranteed cost synchronization of a complex dynamical network via dynamic feedback control," Applied Mathematics and Computation, vol. 218, pp. 6469-6481, 2012.

[25] L. Yi-ping and Z. Bi-feng, "Guaranteed Cost Synchronization of Complex Network Systems with Delay," Asian Journal of Control, vol. 17, pp. 1274–1284, 2015.

[26] D. Ji, J. H. Park, W. Yoo, S. Won, and S. Lee, "Synchronization criterion for Lur'e type complex dynamical networks with time-varying delay," Physics Letters A, vol. 374, pp. 1218-1227, 2010.

[27] Y. Liu, Z. Wang, J. Liang, and X. Liu, "Synchronization and state estimation for discrete-time complex networks with distributed delays," IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, vol. 38, pp. 1314-1325, 2008.

[28] B. Huang, H. Zhang, D. Gong, and J. Wang, "Synchronization analysis for static neural networks with hybrid couplings and time delays," Neurocomputing, vol. 148, pp. 288-293, 2015.