



# Power System Transient Stability Analysis Based on the Development and Evaluation Methods

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## Abstract

A novel method to compute the stability region in power system transient stability analysis is presented. This method is based on the set analysis. The key to this method is to construct the Hamilton-Jacobi-Isaacs (HJI) partial differential equation (PDE) of a nonlinear system, using which we can compute the backward reachable set by applying the level set methods. The backward reachable set of a stable equilibrium yields the stability region of the equilibrium point in power system transient stability assessment. The proposed method is applied to a single machine infinite bus system and a classical two-machine system yielding satisfactory results.

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## 1. Introduction

Power system transient stability [1] is related to the ability to maintain synchronism when subject to a severe disturbance, such as a short circuit on the bus. The resulting system response involves large excursions of generator rotor angles and is influenced by the nonlinear power-angle relationship. Transient stability assessment essentially determines whether the post-fault operating state can reach an acceptable steady-state operating point or not. The conventional method to determine transient stability is to integrate the system equations to obtain a time solution of the system variables, for given system operating points and contingencies. The alternative method is to determine stability directly [2]. The prerequisite of the direct methods is to find the stability region. For a general nonlinear autonomous system, the stability region is defined as the set of all points from which the trajectories start can eventually converge to the stable equilibrium point (SEP) as time approaches infinity [3]. We can determine the stability of a post-fault power system by checking whether the fault-on trajectory at clearing time lies inside the stability region of a desired stable equilibrium point of the post-fault system. If so, it

is guaranteed that the resulting post-fault trajectory will converge to the SEP regardless of the transients of the post-fault trajectory. Therefore, knowledge of the stability region of a SEP is sufficient for the direct transient stability assessment.

In the last three decades numerous efforts have been undertaken to determine the stability region with the goal of power system transient stability analysis. The studies of [4], [5], [6], [7] provide the theoretical foundations for the geometric structure of the stability region. The authors in [6] prove that the stability boundary of a SEP is the union of the stable manifolds of the type one unstable equilibrium points and propose a numerical algorithm to determine the stability region. As the authors indicate, finding the stable manifold of an equilibrium point is difficult. For a planar system, a procedure is suggested in the same paper to numerically determine the stable manifold. However, for higher dimensional systems, the proposed procedure can only find a set of trajectories on the stable manifold. Recently, some algorithms have been developed to approximate the stable manifold of an UEP. For example, in [8], [9]

the Taylor expansion is used to get a quadratic approximation and in [10], [11], the stable manifolds around an UEP are approximated by the normal form technique and the energy function methods [12]. A well-known alternative method called the closest unstable equilibrium point method [2] find a subset of the true stability region and thereby not to obtain the stable manifold of an UEP. The closest unstable equilibrium point method uses the constant energy surface passing through the closest UEP to approximate the stability boundary. It is shown in [13] that the stability region estimated by the closest UEP method is optimal in the sense that it is the largest region within the stability region which can be characterized by the corresponding energy function. However, the closet UEP method can give very conservative results for stability region approximation. In [14], the authors apply the singular perturbation theory to decompose a particular power system into slow and fast subsystems based on the assumption that a power system can be perfectly separated in time-scale. Then the stability region of a SEP is obtained by numerical simulation.

Our paper presents an alternative method to determine the stability region based on reachability analysis. Given a stable equilibrium point of a nonlinear autonomous system (such as a power system), our method can accurately compute the stability region of this SEP, without the information of the unstable equilibrium points.

The paper is organized as following. Some fundamental concepts of the reachable set analysis are introduced in Section 2. In Section 3 a new algorithm to determine the stability region of a SEP is proposed. In Section 4 the algorithm is applied to power system transient stability assessment. The effect of a certain damping ratio on the stability region is also investigated in this section. Section 5 provides conclusion.

## 2. Reachable Set and Its Characters

Reachable sets are a way of capturing the behavior of entire groups of trajectories at once. There are two basic types of reachable sets, depending on whether an initial or a final condition is specified. The forward reachable set is defined as the set of all states that can be reached along trajectories that start in a specified initial set. On the other hand, the backward reachable set is the set of states where trajectories can reach the specified target set. The backward and forward reachable sets are shown in Fig. 1. In section 3, we will make use of backward reachable sets to computer the stability region of a stable equilibrium point of a nonlinear system.

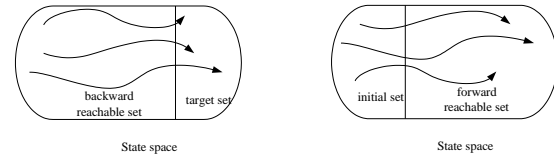


Fig. 1. Illustration of backward and forward reachable sets

One way of describing a subset of states is via an implicit surface function representation. Consider a closed set  $S \subseteq R^n$ . An implicit surface representation of  $S$  would define a function  $\phi(x): R^n \rightarrow R$  such that  $\phi(x) \leq 0$  if  $x \in S$  and  $\phi(x) > 0$  if  $x \notin S$ . In [15], the author formulates the backward reachable set in terms of a Hamilton-Jacobi-Isaacs (HJI) PDE, and proves that the viscosity solution of this PDE is an implicit surface representation of the backward reachable set. This HJI PDE can be solved with the very accurate numerical methods drawn from the level set literature [16].

Consider an autonomous system described by an ordinary differential equation:

$$\frac{dx}{dt} = f(x) \quad (1)$$

where  $x \in R^n$  is the state vector and  $f(x)$  is the vector field.

Suppose  $\phi(x, t)$  is the level function to describe the backward reachability set at time  $t$ .  $\phi(x, t) = 0$  is a surface in  $(n+1)$  dimensional space. The surface  $\phi(x, 0) = 0$  is the boundary of the “target set”, whereas the surface  $\phi(x, t) = 0$  is the boundary of the set of all states  $x \in R^n$  where the target set can be reached in time  $t$  or less. Consider the surface  $\phi(x, t) = 0$  in the  $(n+1)$  dimensional space. For every  $(x, t)$  on this surface the  $\phi$  value is zero. So if we make a small variation along this surface, i.e., move to a neighboring point  $(x + dx, t + dt)$  also lying on the surface, then the variation in value  $\phi$  will be zero:

$$d\phi = \phi(x + dx, t + dt) - \phi(x, t) = 0 \quad (2)$$

$$d\phi = \frac{\partial \phi}{\partial x_1} dx_1 + \dots + \frac{\partial \phi}{\partial x_n} dx_n + \frac{\partial \phi}{\partial t} dt \quad (3)$$

$$= \phi_x^T(x) dx + \phi_t dt = 0$$

From this it follows that,

$$\phi_x^T \frac{dx}{dt} + \phi_t = 0 \quad (4)$$

Substitute (1) to (2), it follows that;

$$\phi_x^T f(x, t) + \phi_t = 0 \quad (5)$$

Thus we obtain the desired PDE and this PDE propagates the boundary of the backward reachability set as a function of time.

Level set methods are a collection of numerical algorithm for approximating the dynamics of moving curves and surfaces. Given a target set defined by an implicit surface function  $\phi(x, t_0)$ , we use level set methods to solve the HJI PDE and thus compute the backward reachable set.

### 3. An Algorithm to Determine Stability Region

Section 2 provides an introduction to the concepts of the reachable sets and their computation using HJI PDE. Here we apply the reachable set analysis for the determination of stability region in power system transient stability assessment. Given a post-fault stable operating point, exist an open neighborhood of it that is contained in the stability region. We pick a sufficiently small  $\varepsilon$ -ball around the SEP as the target set. The backward reachable set with this target set gives the stability region of the post-fault SEP. We check state at the fault clearing time in this backward set in which case system will eventually reach stable operating point. Otherwise, the system will remain unstable. The following algorithm summarizes the procedure to determine the stability region of post-fault power system.

*Step 1:* Form the state space equations of the post-fault power system,  $\frac{dx}{dt} = f(x)$ .

*Step 2:* Find the stable equilibrium point of this autonomous nonlinear system, by solving  $f(x) = 0$  and let  $x^* \in R^n$  be a SEP.

*Step 3:* Specify a  $\varepsilon$ -ball centered at the stable equilibrium point with sufficiently small radius  $\varepsilon$ . Define an implicit surface function at time  $t = 0$  as  $\phi(x, 0) = \|x - x^*\| = \varepsilon$  (6)

Then the target set is the zero sublevel set of the function  $\phi(x, 0)$ , i.e., it is given by:

$$\{x \in R^n \mid \phi(x, 0) \leq 0\} \quad (7)$$

Therefore, a point  $x$  is inside the target set if  $\phi(x, 0)$  is negative, outside target set if  $\phi(x, 0)$  is positive, and on the boundary of the target set if  $\phi(x, 0) = 0$ .

*Step 4:* Propagate in time the boundary of the backward reachable set of the target set by solving the following HJI PDE:

$$\phi_x^T f(x, t) + \phi_t = 0 \quad (8)$$

With terminal conditions:

$$\phi(x, 0) = \|x - x^*\| = \varepsilon \quad (9)$$

The zero sublevel set of the viscosity solution  $\phi(x, t)$  to (6), (7) is the backward reachable set at time  $t$ :

$$\{x \in R^n \mid \phi(x, t) \leq 0\} \quad (10)$$

*Step 5:* The backward reachable set of the  $\varepsilon$ -ball around the stable equilibrium point is computed using software tool [18]. It is always contained in the stability region of the stable equilibrium point. And  $t$  goes to infinity, the backward reachable set approaches the true stability region. If the stability region is bounded, the level set based numerical computation of the backward reachability set eventually converges to the stability region within a finite computation time.

### 4. Examples

#### A) A Single-Machine-Infinite-Bus Model

The classical single-machine-infinite-bus model of power systems is shown in Fig. 2. The system model is given as follows:

$$\frac{d\delta}{dt} = \omega \quad (11)$$

$$\frac{d\omega}{dt} = \frac{1}{M}(P_m - P_e^M \sin\sigma - D\omega) \quad (12)$$

Here,  $\delta$  is the machine rotor angle and  $\omega$  is the angular velocity of the rotor. Suppose the inertial constant  $M = T_j / \omega_0 = 0.026 \text{ s}^2 / \text{rad}$ , and  $P_m = 1.0$  per unit,  $P_e^M = E'U/X = 1.35$  per unit.

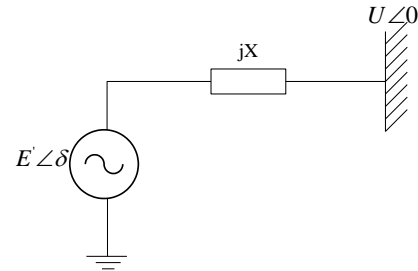


Fig. 2. A single-machine-infinite-bus model

From the system equations (11) and (12), and the chosen parameter values, the point (0.8324, 0) is identified to be the unique stable equilibrium point of this system. We set the target set as  $\sqrt{(\delta - 0.8324)^2 + \omega^2} \leq 0.1$ , and choose the damping coefficient  $D$  to be 0.12 s/rad. On a standard laptop, the backward reachable set computation converges. The stability region lies inside the solid line drawn in Fig.3. From this figure, we can conclude that if the post-fault initial condition of the state variables is inside the stability region, the trajectories will converge to the stable operating point. And if the initial condition is outside the stability region, the system will remain unstable. We validate our result

by drawing the corresponding phase portrait, using some time domain simulation of sample trajectories, from which we can see that our method can precisely determine the stability region.

In addition, when the damping coefficient  $D$  is increased, the stable equilibrium point remains the same as  $(0.8324, 0)$ . For  $D = 0.15$ , we compute the stability region as shown in Fig. 4. The computation terminates in 5.14 seconds on a slandered laptop. The figure clearly shows that when  $D$  is increased, the size of the stability region also increases. The observation is validated by time domain simulations. Again, figures 5 and 6 give the time domain responses of the rotor angle and velocity for an initial condition  $(\delta_0, \omega_0) = (-5, 15)$  when  $D$  is 0.12 s/rad and 0.15 s/rad, respectively. Fig. 6 shows the trajectories eventually settle to the post-fault stable operating point. However, when  $D$  is 0.12 s/rad, the system loses stability for this initial condition as shown in Fig. 5. This is not unexpected since a large  $D$  reduce implies a larger stability region.

#### B) A Classical Two-Machine Model

A simple power system containing two generators is shown in Fig. 7. The classical model of the post-fault system is as following:

$$\frac{dx_1}{dt} = x_2 \quad \frac{dx_2}{dt} = -0.7143x_2 + 0.234 - 0.0633 \sin(x_1 + 0.0405) - 0.582 \sin(x_1 + 0.4103) \quad (13)$$

where  $x_1 = \delta_1 - \delta_2$  is the angular difference between the two generators, and  $x_2 = \omega_1 - \omega_2$  is the angular velocity difference between the two generators. In this system, the origin is a stable equilibrium point,  $(2.393539, 0)$  and  $(-3.889646, 0)$  are saddle points. This can be confirmed by linearization. Most of the previous calculation of the stability region of this two-machine system is of the form shown in Fig. 8 [17]. The computation by the present method is shown in Fig. 9. Here, we chose the radius of  $\mathcal{E}$  ball to be 0.1 and after 25.8 seconds' computation time we obtained the stability region. This figure also shows the phase portrait of the system. It is clear that the computed stability region closely matches the result of the phase portrait.

## 5. Conclusion

A novel method for computing the stability region of nonlinear system, such as power systems is presented in this paper. The proposed method has the following advantages:

- It computes the stability region accurately. For large systems, the computation may be stopped after a certain time to get a sub region.

- It is easy to implement. We only need to form the mathematic model of the post-fault power system and identify the stable equilibrium point. After then, we can use level set methods to compute the stability region as a backward reachable set.

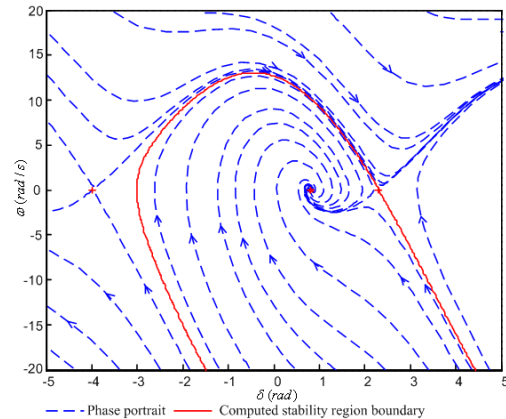


Fig. 3. Stability region and phase portrait for  $D=1.2$  s/rad

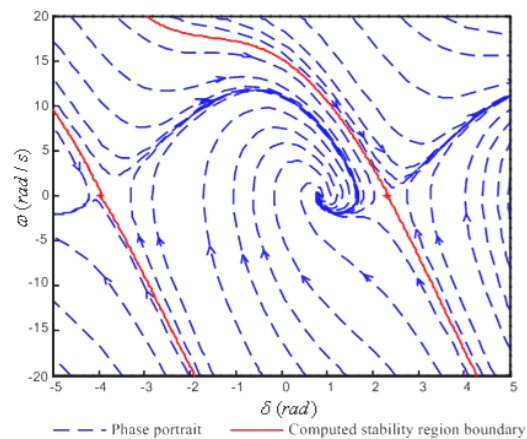


Fig. 4. Stability region and phase portrait for  $D=1.5$  s/rad

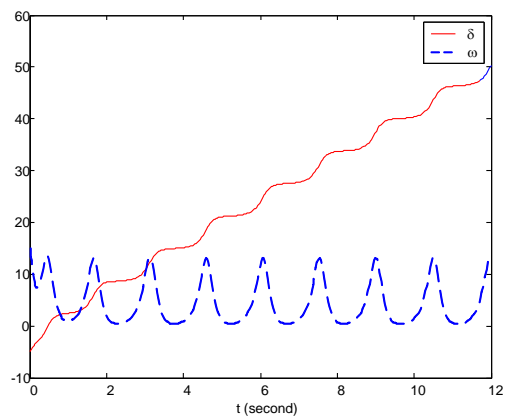


Fig. 5. Time domain simulation when  $D$  is 0.12 s/rad

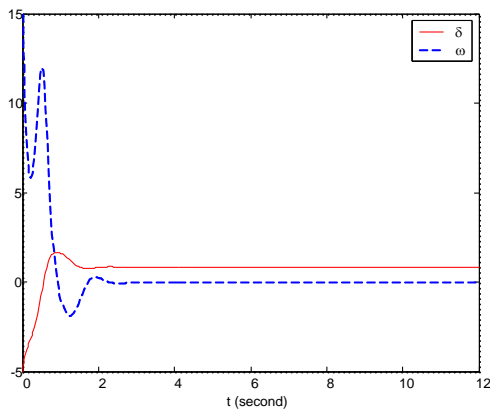


Fig. 6. Time domain simulation when  $D$  is 0.15 s/rad

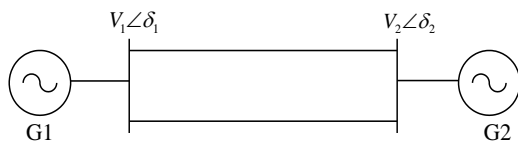


Fig. 7. Two-machine power system.

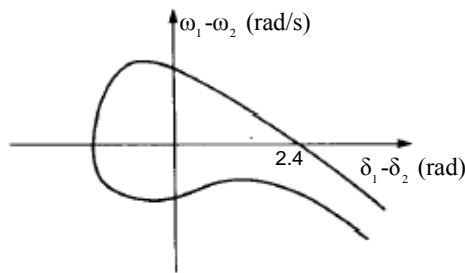


Fig. 8. Prior approximation of the stability region

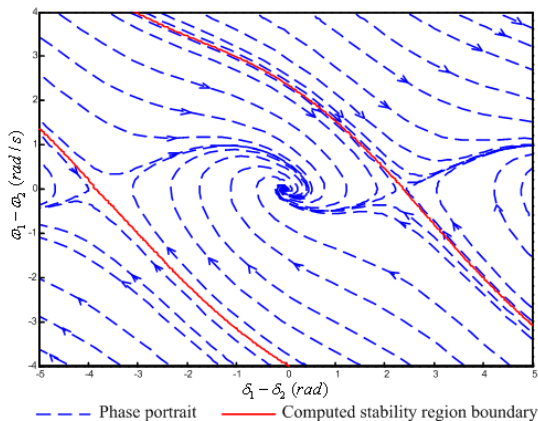


Fig. 9. Our computation of stability region and phase portrait

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