Power Quality Monitor Placement Using a Tri-level Approach

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Abstract

Finding minimum number of connecting lines is as important as locating power quality monitors (PQMs) for full observability of power system. Therefore, a PQM placement method should determine both optimum buses and lines, since utilities can make better decisions for monitoring of power system with this information. This paper attempted to propose a new method to locate the PQMs at various unobservability depths. In this method, the problem of placement is solved on three levels, taking into account the limited number of channels required for measuring each bus. At the first level of this method, the optimum combination of zero injection buses (ZIBs) is achieved. At the second level, different combinations of connecting lines are produced, while the third level determines the best location of monitors in different nonobservability depths. In addition, the third level identifies the critical buses and their effects on the number and location of monitors. Moreover, the best location for installing monitors and the number of channels required by equipment in each depth will be specified. The results of applying the new method on 14-bus IEEE network demonstrate the ideal performance.

Keywords: Power quality monitor placement; Measurement channel; Depth of unobservability; State estimation; Optimization

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1. Introduction

Nowadays, accessibility and reliability of power quality are highly crucial, because the equipment used in modern industry rely on semiconductor and microprocessors which are extremely sensitive to power disturbances. The power quality disturbances such as overvoltage or undervoltage, harmonics, transient phenomena etc. could affect consumers. In fact, the electrical equipment that is used today are so sensitive that poor quality power delivery can lead to malfunction of the equipment as well as technical/economic issues [1]. The power consumers, particularly the industrial consumers, require that power quality be delivered at a reliable and appropriate level.

The first step in evaluation of power quality is monitoring. The task of monitoring in power grids is not a new subject matter, but the issue of monitoring power quality phenomena has been discussed only over recent years. There are a number of power quality phenomena that cannot intrinsically be detected through routine monitoring equipment. Hence, the monitors employed for that purpose are deemed expensive instruments. Ideally, it is critical to install a monitor in all buses of a grid. However, considering the costs of monitors and data processing, it will not be cost-effective. There have been numerous methods proposed so far to determine the optimum number and location of power quality monitors. In general, the placement methods for power quality monitors can be divided into four categories: monitor coverage area (MRA), graph theory (GT), multivariate regression (MVR) and coating and packaging (CP) [2].

In 2003, a new MRA-based method was introduced to determine the optimum location of monitors. MRA represents an area of grid visible from the position of a monitor. According to this definition, the measurement device will record the fault if it occurs inside the MRA; otherwise, it will not be recorded [3]. [4] is one of the first studies in this field adopting the branch-and-bound algorithm. This algorithm divides the search space into smaller spaces, but may obtain incorrect solutions by selecting the wrong space in the early stages. Hence, it was replaced with algorithms demonstrating better performances. For instance, [5] adopted the quantum-
inspired binary firefly algorithm (QBFA) in order to speed up convergence and solving the localization problem in a multi-objective procedure. Although it is convenient to adopt the MRA matrix to locate monitors and detect any voltage sags, it will not be desirable for radial distribution grids. In 2008, Dong presented an algorithm based on graph theory in an effort to monitor voltage sags in power grids. In this method, the power grid is displayed by a simple graph and then the grid coverage matrix is achieved. Moreover, it is essential to specify weight factors for all circuit elements. Therefore, this method is ideal for displaying the relationship between elements and real points in the power grid. In addition, the graph theory needs to determine a rooted tree where there is a parent-child relationship. Since there might be fault in determining this relationship in the transmission grids, the GT-based methods are suitable only for radial distribution grids. In 2011, a new method was presented based on multivariate regression model, involving static indices Cp and Rp in localization of PQMs. In this method, all data related to single phase to ground (LG), double phase to ground (LLG) and three phases to ground (LLL) faults were collected on each bus. Then, the correlation coefficient (CC) is calculated to indicate the relationship between buses during the disturbances. At the next stage, the two buses with the highest CC values are detected as the most sensitive buses across the grid. The voltages of the two buses are considered as an independent variable in the MVR model developed to estimate the voltages of other buses.

There are numerous papers employing the CP-based methods, where, the constraints of the problem ensure that all state variables (bus voltages and line currents) are observable at least by one monitor. The observability of a system depends on state equations, which can be formulated based on Ohm’s Law and Kirchhoff’s Current Law (KCL). In the new method involves multi-objective placement, where the results suggest that reduction in the number of monitors reduces the final cost of installation, while mitigating the information redundancy measured by monitors. In this method, the optimization problem is solved by three connection matrices formed based on bus interconnections, which are then used to form the system’s density matrix similarly employed the MEAT algorithm to minimize the costs of installing monitors and maximize the redundancy factor. Moreover, the constraint was obtained based on the circuit topology regardless of load constraints. In [18], the problem is solved through a branch-and-bound algorithm, where the cost of installation is specified according to the number of lines passing each bus. Ref. [19] proposed a method for monitoring power quality in distribution systems based on the p-median model. In this paper, the current techniques were initially adopted to obtain the lowest number of monitors for system observability. Then, the modified P-median model was used. This model involves a limitation in dealing with localization of monitors based on importance of loads.

In most relevant, the problem of placement has been explored from the perspective of grid topology serving to improve the solution. However, there are several factors and constraints affecting the number and location of monitors. One of the factors potentially contributing to placement of power quality monitors is the number of measurement channels in each monitor. Since the number of measurement channels in each device is effective on the marginal cost of purchase and installation of PQM, the involvement of the above parameters can lead to more efficient option than conventional methods. The grid zero injection buses can be another factor contributing to optimum placement of monitors. Since the KCL applies to these buses, they can be integrated with their neighboring buses in an effort to mitigate the final cost of installation. In addition, it is essential to install a monitor in a number of buses for each grid known as critical buses. Thus, it is essential to provide a strategy to identify critical buses.

This paper intended to propose a new method for finding the optimum location of power quality monitors in an effort to curtail the final cost of purchase and installation as well as the estimation error of voltage phasor in unobservable buses. Moreover, the new method covered the effects of zero injection buses and the number of measurement channels in each device. In this procedure, it can be specified how many channels should be there in the monitor installed in each bus and which connection lines should be measured. In addition, the state estimation, and more specifically, the state estimation index (SEI) can be employed to identify critical buses of a grid and include it in the placement procedure. This approach involves three levels specifying, 1) the optimum combination of zero injection buses, 2) different combinations of buses connections, and 3) the best location of monitors at different depths of unobservability. It should be noted, however, the ultimate objective of the new method is to minimize the estimation error in unobservable buses while mitigating the final costs of purchasing and installing power quality monitors.

2. Placement of Power Quality Monitors

The problem of PQMs placement can be formulated as a classic combinatorial optimization problem. In [17], the common method of finding the optimum location of power quality monitors has been represented and used. This method involves a binary optimization problem, where the equations are as follows.
\[
\min f(x) = \sum c_i x_i = C^T \cdot X
\]

subject to: \(DX \geq b\)

In (1), Matrix \(C^T\) represents the cost of installing PQM in each bus. This matrix is \(n \times 1\) order, where \(n\) is the number of buses in the grid. \(X\) represents a binary matrix with \(n \times 1\) order. After solving the above problem, the elements of matrix \(X\) are 0 and 1, where zero elements represent that there is no need to install PQM at the corresponding buses, while unity elements show the buses that their data is required for achieving full observability of the system. Equation (2) provides assurance that each required parameter, i.e. bus voltage and line current, are directly or indirectly observable at least by one monitor. In this Equation, \(b\) is a column vector with elements 1 as many as the number of rows in matrix \(D\). Moreover, \(D\) is a density matrix [15].

A density matrix can be formed first by obtaining the grid incidence matrix (\(A\)) with dimensions of \(m \times n\) (\(m\) represents the number of state variables and \(n\) is the number of buses). In this matrix, column \(k\) indicates the buses; row \(r\) represents the state variable \(r\) (bus voltage or line current). Each member of matrix \(A\) is defined as follows:

\[
A(r,k) = \begin{cases} 
1, & \text{if } r \text{ is observed by PQM } k \\
0, & \text{otherwise}
\end{cases}
\]

In fact, matrix \(A\) can be divided into two sub-matrices, one of which concerns the observability of bus voltages (\(A_v\)) while the other concerns the observability line currents (\(A_l\)). The sizes of these sub-matrices are \(nxn\) and \(lxn\), where \(l\) represents the number of lines and \(n\) is the number of buses. Therefore, matrix \(A\) can be formulated as follows:

\[
A = \begin{bmatrix} A_v \\ A_l \end{bmatrix}
\]

Each member of matrix \(A_v\) is defined as follows:

\[
A_v(r,k) = \begin{cases} 
1, & \text{if } r = k \text{ or buses } t \text{ and } k \text{ are connected} \\
0, & \text{otherwise}
\end{cases}
\]

Similarly, matrix \(A_l\) can be formulated as follows:

\[
A_l(r,k) = \begin{cases} 
1, & \text{if line } r \text{ is connected to busbar } k \\
0, & \text{otherwise}
\end{cases}
\]

In fact, a bus voltage becomes observable when a monitor is installed in that bus or one of the adjacent buses. Moreover, a line current becomes observable when a monitor is installed in every bus on both sides of the line.

Matrix \(A\) is adopted to define matrices \(B_j\) and \(B_k\) as described in [15]. At the next stage, these matrices will be used to form matrix \(D\).

\[
B_j(r) = \begin{cases} 
A(j), & \text{if } r \text{ represents } i_j \text{ and busbars } j \text{ and } k \text{ are connected} \\
0, & \text{otherwise}
\end{cases}
\]

\[
B_k(r) = \begin{cases} 
A(k), & \text{if } r \text{ represents } i_k \text{ and busbars } j \text{ and } k \text{ are connected} \\
0, & \text{otherwise}
\end{cases}
\]

In forming matrices \(A\), \(B_j\) and \(B_k\), the following points should be considered:

1) \(B_j\) and \(B_k\) are defined only for current state variables, while assuming zero for all other variables.

2. When formulating these matrices, the state variables should be classified as follows:

The voltages and currents are both arranged in ascending orders.

In solving the problem of localizing PQM placement, matrix \(D\) is obtained as follows:

\[
D = \begin{bmatrix} A_{(ln)} \\ A_{(Lm)} \\ A_{(km)} \end{bmatrix}
\]

Where \(A_{(ln)}\) part of the incidence matrix from row 1 to \(n\), \(A_{(Lm)}\) is part of the incidence matrix from row \(L\) to \(m\). \(B_{j(Lm)}\) and \(B_{k(Lm)}\) are parts of these matrices from row \(L\) to \(m\). Hence, the dimensions of the density matrix \(D\) are \((n + 2L) \times n\).

3. Proposed Method

In solving the problem of placement, this paper proposed a tri-level model, where the voltage phasor estimation error at unobservable buses and the total cost of PQM installation are minimized as the main objectives. The first level of the model determines the optimum combination of ZIBs, the second level produces different combinations of connecting lines, and the third level outlines where the monitors should be installed for different combinations of connection lines. It should be noted, however, the new method involves the PSO algorithm in the first and second levels and the integer linear programming (ILP) in the third level. Fig.2 illustrates the overall flowchart of the proposed method, while Fig.3 provides the flowchart of the second and third level of the method.

Level 1: As stated previously, the first level of the new method involves the PSO algorithm serving to find the optimum combination of zero injection buses. The ZIBs effect on PQMs placement problem is considered using the method defined in [20]. In the
proposed method, level 1 considers the number of algorithm variables equal to the number of zero injection buses. Moreover, the minimum values of all variables are 1 and maximum values are equal to the number of buses which are connected to each ZIB. In order to gain a better understanding, let us assume a 5 bus system which is shown in Fig.1. In this grid, if bus 4 is the only zero injection bus in the grid, then there will be 1 variable whose minimum and maximum are 1 and 3, respectively. At this level, initial population m/j is generated given the buses where ZIBs can be integrated. Each population represents the integration of zero injection buses with one of their adjacent buses. Finally, each member of the initial population is analyzed as input at the second level of the problem.

Fig. 1. Example power system

Level 2: With respect to the number of channels and how ZIBs are integrated in the first level, the second level forms the decreased incidence matrix for various combinations of lines in each member of the initial population. The decreased incidence matrix refers to a matrix in which integration of zero injection buses takes place. It should be noted that in order to consider the effect of channels on PQM placement problem, the method defined in [21] is used.

At this level, the number of algorithm variables is equal to the number of buses, other than zero injection buses. Furthermore, the minimum values all variables are 1, while the maximum value of each variable is equal to the number of connected buses. It should be noted, however, this level does not consider the zero injection buses as variables, because the first level of the new model has been already dedicated to these buses and how they integrate. At this level, the decreased incidence matrix is formed for various combinations of lines with the number of members in the initial population.

Level 3: In the third level of this model, the density matrix for each decreased incidence matrix at an unobservability depth of d, which is defined in [22], is formed. Equations (1) and (2) and the linear programming are adopted to achieve the corresponding location of each density matrix for installation. Obviously, the placement of monitors creates unobservable areas in the network, which involve one or more unobservable buses depending on the unobservability depth. Hence, the placement of monitors yields a set of unobservable buses and a set of observable buses. After placement of monitors at this level, a network model defined by the admittance matrix has been adopted to estimate the voltage of unobservable buses through observable buses [23].

After the state estimation, the critical buses of the grid are examined. The critical buses can be identified by state estimation index (SEI). The SEI represents the difference between the estimated values and the results of power flow, indicating the voltage estimation error. After estimating the state for each member of the population, this stage identifies the buses whose estimation errors exceed that of SEI. Moreover, a critical bus will indicate the greatest difference from SEI, while placement is resumed by adding the following constraint to the placement problem [24].

\[ A_{eq}X = b_{eq} \]  \hspace{1cm} (10)

The equation constraint expressed in (10) provides assurance that at least one monitor is inserted in each critical buses. In this Equation, \( A_{eq} \) is a binary matrix with dimensions \( n \times 1 \), where \( 1 \) for element \( i \) indicates that at least one monitor is inserted in bus \( i \). \( b_{eq} \) represents the number of non-zero elements in matrix \( A_{eq} \).

Fig. 2. Flowchart of the proposed method
In the newly proposed method in this paper, the procedure of checking critical buses continues until the estimation error in all buses become lower than SEI, while the number of critical buses is progressively added at each stage. Then, the estimated values (which are all lower than SEI) are stored in the EE matrix according to Equation (11). In addition, the cost of installing monitors for each member of the initial population is calculated according to (13) and stored in the Cost matrix.

\[
EE = \begin{bmatrix}
e_{1,1} & L & e_{1,2} & L & e_{1,N_B} \\
M & L & M & L & M \\
e_{N_B,1} & L & e_{N_B,2} & L & e_{N_B,N_B}
\end{bmatrix}
\]  (11)

In this Equation, \( e_{i,k} \) represents the estimation error at bus \( k \) for particle \( i \). \( N_B \) is the number of buses in the network, and \( N_{pop} \) is the number of members in the population. It should be noted, however, the estimation error is assumed to be zero in buses where monitors have been installed. Then, the estimated values are averaged and the results are stored in matrix \( EE_{avg} \) according to Equation (12).

\[
EE_{avg} = \begin{bmatrix}
e_{1,avg} & L & e_{2,avg} & L & e_{N_B,avg}
\end{bmatrix}
\]

\[
\text{cost}_i = \left( N_{pop} \times P_{PQM} \right) \times \left( N_{ch} \times P_{ch} \right) + \left( N_{uch} \times P_{uch} \right) \quad i = 1, 2, ..., N_{pop}
\]  (13)

\[
\text{Cost} = \left[ \text{cost}_1, \text{cost}_2, ..., \text{cost}_{N_{pop}} \right]
\]  (14)

In Equation (13), \( \text{cost}_i \) represents the installation cost for each member of the initial population. \( N_{pop} \) is the number of monitors installed, \( N_{ch} \) is the total channels used except those used to measure bus voltage, \( N_{uch} \) is the sum of unused channels and \( N_{row} \) is the number of initial population. Furthermore, \( P_{PQM}, P_{ch} \) and \( P_{uch} \) are the prices related to the installation of a PQM, its used channels and unused channels, respectively.

In this paper, the cost of installing each PQM has been divided into two fixed and variable costs. The latter includes the costs of monitor, while the former includes the costs associated with each additional channel other than that used to measure bus voltage.

At the next stage, the stored costs and estimation error values for each individual member of the population will be normalized through Equations (15) and (16).

\[
\frac{\text{cost}_i}{\text{cost}_{max}} = \frac{\text{cost}_i}{\text{cost}_{max}}
\]  (15)

\[
\frac{EE_{avg}}{SEI} = \frac{EE_{avg}}{SEI}
\]  (16)

In Equation (15), \( \frac{\text{cost}_i}{\text{cost}_{max}} \) represents the normalized value of installation cost for each member of the population, while \( \frac{EE_{avg}}{SEI} \) represents the maximum cost of placement in this population. In Equation (16), \( \frac{EE_{avg}}{SEI} \) represents the normalized values of estimation error for each member of the population.

Then, the corresponding values of normalized cost and estimation error are summed up and stored in the Fitness matrix according to (17).

\[
\text{Fitness} = [\text{fitness}_1, \text{fitness}_2, K \text{fitness}_{N_{pop}}]
\]  (17)

Each member is obtained according to the following equation:

\[
\text{fitness}_i = \frac{EE_{avg} + \text{cost}_i}{SEI} \quad i = 1, 2, ..., N_{pop}
\]  (18)

Finally, the lowest value in the Fitness matrix is selected as the best solution, while the initial population is upgraded and a new population is generated. This process is repeated according to the number of algorithm iterations. At the end, the best location of monitors for installation at various invisibility depths will be obtained in a way to include the effect of zero injection buses and critical buses in placement, while achieving minimal installation cost and minimal estimation error.

4. Results and Discussion

In this paper, the proposed method for PQM placement is applied on IEEE 14-bus power system. In this system, bus 7 is the only zero injection buses which is connected to buses 4, 8 and 9. It should be noted that, in all simulation performed in this section, the following cases are considered:

- Only one PQM is installed at each bus.
- The cost of each monitor, and each used and unused channel is assumed to be 15000, 3000 and 500 $ respectively.
- SEI is considered to be 0.04 pu and 5 degrees for amplitude and phase angle of each voltage respectively.

In order to consider the effect of the number of channels on PQM placement, at first, it is assumed that just 2-channel PQMs are used. Table 1 shows the number of monitors (\( N_{pop} \)), used and unused channel (\( N_{ch} \) and \( N_{uch} \) respectively) obtained from the proposed method for different depths of unobservability. According to row 1 Table 1 in zero depth of unobservability, seven PQMs are required for complete observability whether the effect of ZIB (bus 7) is considered or not. Also without considering the effect of ZIB, the number of used channel is 7. When the effect of ZIB is considered, the number of used channel is 6. Furthermore, by increasing the depth of unobservability, the number of PQMs, which are required for complete observability, decreases.
Table 2. shows the results when it is assumed that all PQMs have 3 channels. As this table shows, similar to the results of Table 1, the number of PQMs required for complete observability of the network decreases with increasing the unobservability depth. Table 3. and Table 4. show the results when all PQMs have 4 and 5 channels respectively. From these tables it is clear that as the number of channels increases, the number of PQMs decreases, but the number of unused channel increases. This is because by using PQMs with higher number of measuring channels more unobservable buses can be observed by devices installed at the observable buses.

Fig.4 shows the total cost of installation for different depths of unobservability in the case which the effect of ZIB is considered. It can be seen that, by increasing the depth of observability, the total cost has reduced. Furthermore, it is clear from Fig.4 that the minimum cost is achieved when 3-channel PQMs are used in depth 4.

5. Conclusion

This article presented a new method for the optimal placement of PQMs in a power network. In this method, placement problem was divided into three levels to specify the best position for installing devices and lines whose currents should be measured at various depths of unobservability. To do so, the estimation error of amplitude and phase angle of voltages at unobservable buses and the effect of zero injection buses were considered in the placement problem. The results proved that fewer devices are required for complete monitoring of the network with increasing the depth of unobservability. On the other hand, final cost reduces at any depth with increasing the number of channels of monitors. Then cost will increase with the increasing number of unused channels. Finally, it is possible to find the optimal place for installing monitors in a way that the error of estimating power quality parameters in other buses is low through considering the above items and using the proposed method.

![Flowchart](image-url)
Table 1. Optimal PQM placement for IEEE 14-bus system with considering 2 channels for each PQM

<table>
<thead>
<tr>
<th>Depth</th>
<th>N_{PQM}</th>
<th>N_{bus}</th>
<th>Cost</th>
<th>Bus</th>
<th>N_{PQM}</th>
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<th>N_{sag}</th>
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Table 2. Optimal PQM placement for IEEE 14-bus system with considering 3 channels for each PQM

<table>
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<th>N_{bus}</th>
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Table 3. Optimal PQM placement for IEEE 14-bus system with considering 4 channels for each PQM

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Table 4. Optimal PQM placement for IEEE 14-bus system with considering 5 channels for each PQM

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<th>N_{bus}</th>
<th>Cost</th>
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Fig. 4. The total cost of PQM placement for IEEE 14-bus system with considering ZIB.

References


