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A Novel Qualitative State Observer

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Abstract

The state estimation of a quantized system (Q.S.) is a challenging problem for designing feedback control and model-based fault diagnosis algorithms. The core of a Q.S. is a continuous variable system whose inputs and outputs are represented by their corresponding quantized values. This paper concerns with state estimation of a Q.S. by a qualitative observer. The presented observer in this paper uses a non-deterministic automaton as its qualitative model and estimates quantized values of the system state. Observer inputs are on-line measured input and output signals of Q.S. The previous proposed qualitative observers use dynamics of the continuous variable system of Q.S., whereas in this paper, the qualitative observer model is built by a quantitative observer. The main theorem of the paper shows that if the parameters of quantitative observer and sampling time are chosen correctly, then qualitative estimation error will be uniformly ultimate bounded, i.e. it will converge to a bounded convex set. In addition, simulation results show that reducing bounds of the convex set, results in less additional generated spurious states.

Keywords: Qualitative observer, quantised system, nondeterministic automaton, spurious states
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1. Introduction

An interesting area of recent researches in hybrid systems has been devoted to the Fault Detection and Isolation (F.D.I.) for hybrid systems. Hybrid systems involve the interaction of time-driven dynamics with event-driven dynamics and provide a convenient framework for modeling systems in a wide range of engineering applications [15].

The main part of a model-based FDI procedure is the generation of residuals which reflect the discrepancy between two different faulty modes. One of the general approaches for residual generation is observer-based approaches [4]. Due to mixed behavior of hybrid systems, using state observers for these systems is more complicated and some solutions are proposed in literature [1, 13]. In some of the other solution approaches, the system is considered as Q.S. whose qualitative model is used to build qualitative observer [2, 3, 10, and 11]. In comparison with these approaches, Fig. 1 illustrates the idea of our qualitative observer for Q.S. The qualitative state observer estimates consistent states with observed input and output sequences. A non-deterministic automaton (N.D.A) is used for discrete-event description of the qualitative state observer. The N.D.A describes a relation between qualitative input sequences (denoted by $[\boldsymbol{U}]$) and qualitative output sequences (denoted by $[\boldsymbol{Y}]$) (c.f. section 2 for further details). This observer must have a complete model. A qualitative model is complete when it can generate all output sequences that the quantized system may generate for all input sequences [11].

The qualitative observers presented in the literature, use different qualitative models such as N.D.A., stochastic automaton, timed automaton and Petri nets [5, 7, 8, 10].In all of these observers, dynamics of the continuous variable system of Q.S. is used to build the qualitative model of the observer. In this paper, the qualitative model of the observer is constructed based on an asymptotically stable observer. By this modification, it is shown that qualitative estimation error (Q.E.E.) can be converted to a bounded convex set. In [3], it is shown that if the model of qualitative observer is complete, then the actual system state exists in the set of estimated states. But convergence of Q.E.E. is not discussed completely. In this paper, it is shown that if the quantitative observer is asymptotically stable and the sampling time of quantizer is chosen properly, then Q.E.E of qualitative observer converges to a bounded convex set. In addition, simulation results show that the bounds of the convergence set can be reduced by varying the design parameters of the quantitative observer and sampling time. Also, reduction of Q.E.E. results in less additional generated spurious states, which in turn, increases the performance of F.D.I algorithms for hybrid systems.

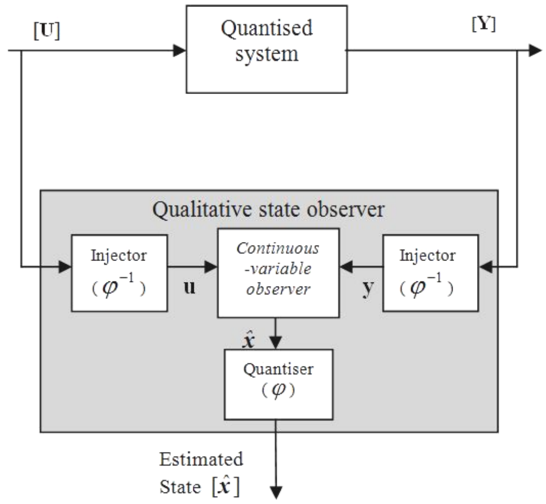

Fig. 1. Qualitative state observer.

Since the mentioned method is based on qualitative modeling, qualitative model representation of Q.S. and the qualitative state estimation is described briefly in section 2. In section 3 a brief review of Luenberger state observers is given and then some basic definitions and conditions for convergence of Q.E.E are presented. For more illustration, the proposed observer is applied on a two-tank system in section 4. By simulation results, the effect of sampling time and quantitative observer parameter variations on Q.E.E is investigated.

2. Qualitative Observer

Since qualitative models are used for qualitative observer, this section gives a brief description of the qualitative modeling of a continuous-variable discrete-time system [6]. Furthermore, the qualitative state observation task is given at the end of this section. References [3, 6, 11], are useful for further details.

The core of a Q.S. is a continuous-variable discrete-time system which is defined as follows (c.f. Fig. 1):

$$\boldsymbol{x}(k+1)=\boldsymbol{f}_C(\boldsymbol{x}(k),\boldsymbol{u}(k)\ ,\ \boldsymbol{x}(0)=\boldsymbol{x}_0$$
$$\boldsymbol{y}(k)=\boldsymbol{g}_C(\boldsymbol{x}(k),\boldsymbol{u}(k)), \tag{1}$$

where $\boldsymbol{x}(k)\in\mathbf{R}^n$ denotes the system state, $\boldsymbol{u}(k)\in\mathbf{R}^m$ denotes the system input , and $\boldsymbol{y}(k)\in\mathbf{R}^r$ denotes the system output.

The state quantizer introduces a partition of the state space $\mathbf{R}^n$ into a finite number of disjoint sets $Q_x(i)$, $i\in N_x=\{0,1,\dots,N\}$. The qualitative value of the state $\boldsymbol{x}(k)$ at time *k* is given by the index *i* of the set $Q_x(i)$ to which the state belongs

$$[\boldsymbol{x}(k)]=i \Leftrightarrow \boldsymbol{x}(k)\in Q_X(i) \tag{2}$$

In a similar manner the output quantizer introduces a partition of the output space $\mathbf{R}^r$ into a finite number of disjoint sets $Q_y(j)$, $j\in N_y=\{0,1,\dots,R\}$. $Q_y(i)$ denotes the set of outputs $y\in\mathbf{R}^r$ with the same qualitative value i, i.e.

$$[\boldsymbol{y}(k)]=i \Leftrightarrow \boldsymbol{y}(k)\in Q_Y(i) \tag{3}$$

Similarly, the input space partitioning results in discrete set $Q_u(l)$, $l\in N_u=\{0,1,\dots,M\}$ and the set of faults is denoted by $N_f=\{f_0,f_1,\dots,f_s\}$. For all partitions, it is assumed that sets $Q(i)$ $(i\neq 0)$ are bounded and the remaining part of the signal space is $Q(0)$. For example, for state space quantizer :

$$Q_x(0)=\mathbf{R}^n\setminus(\bigcup_{i=1}^{N}Q(i)) \tag{4}$$

The injector φ^{-1} shown in figure 1 maps the qualitative value $[\boldsymbol{u}(k)]$ to a continuous value. Since the qualitative value $[\boldsymbol{u}(k)]$ represents a set, each of its elements can be assigned to output of the injector, that is $\varphi^{-1}([\boldsymbol{u}])\in Q_u$.

The qualitative model describes a relation between all possible qualitative input sequences and all corresponding qualitative output sequences of the Q.S.

$$[U]^T=([\boldsymbol{u}(0)],[\boldsymbol{u}(1)],\dots[\boldsymbol{u}(T)]) \tag{5}$$

$$[\boldsymbol{Y}_S]^T \;\square\; ([\boldsymbol{y}(0)],[\boldsymbol{y}(1)],\dots[\boldsymbol{y}(T)]) \tag{6}$$

Both of sequences of Eqs. (5) and (6) are indexed here with the time horizon T. An adequate model which generates the set $[\boldsymbol{Y}_M]^T$ of qualitative output sequences has to fulfil the condition for all qualitative input sequences and faults and all sets of initial states. Such a model is called a complete model [1, 10].

$$[\boldsymbol{Y}_M]\supseteq[\boldsymbol{Y}_S] \tag{7}$$

In this paper non-deterministic automaton is used as qualitative model. A non-deterministic automaton is defined as:

$$N\,(N_Z\ ,N_V\ ,N_W\ ,L,\mathbf{z}(0)) \tag{8}$$

In which, z is the state vector of the non-deterministic automaton, N_z , N_W and, N_V denote the sets of qualitative values of states, qualitative values of outputs and qualitative values of inputs, respectively.

The behavioral relation describes all possible successor states $\mathbf{z}(k+1)$ and outputs w_k of the system for a given current state $\mathbf{z}(k)$ and input v_k at time instant *k*.

$$L : N_Z \times N_Z \times N_V \times N_W \rightarrow \{0,1\} \tag{9}$$

Such a model is a generator of the set of all I/O sequences given the initial state $\mathbf{z}(0)$ and hence, a compact representation of the system behavior. In L, a one indicates that a transition is possible while a zero means that it is not [11]. It is shown that this qualitative modeling approach provides a complete model that is needed in qualitative observation task [3, 11]. Note that, in this paper the qualitative model is used for the state observer rather than the Q.S.

The previous qualitative observers (which called pure qualitative observers in this paper) find all possible qualitative states which are consistent with observed qualitative input and output sequences [3, 11]. Since the inputs and outputs can only be measured qualitatively, and since a quantitative observer is used in our approach, the necessary signals are obtained by injectors (c.f. Fig. 1). By our approach, we are able to adjust parameters of quantitative observer; such as error convergence rate, to satisfy our desired specifications. Since the overall obtained configuration for the proposed observer (the gray box in Fig. 1) forms a Q.S., it is replaced by a qualitative model. This qualitative model is a N.D.A and it is qualitative abstraction of the designed quantitative observer rather than the continuous variable system. Convergence of the qualitative estimated states is an important problem in the observation task. Hence, in the next section, this problem is investigated and we present coefficient conditions that imply convergence of Q.E.E.

3. Estimation Error Convergence

In this section the conditions for convergence of Q.E.E are derived. In our approach, at first step we design a quantitative state observer. For simplicity in this paper, the continuous-variable system is considered as a linear system; that *A*, *B* and *C* are state parameters. A Lunberger observer is used for quantitative observer as [14],

$$\begin{aligned} x^{\cdot} &= \mathrm{A}x + \mathrm{Bu}, y = Cx \\ x^{\wedge\cdot} &= \mathrm{A}x + \mathrm{Bu} + K(y - y^{\wedge}), y^{\wedge} = Cx^{\wedge} \end{aligned} \tag{10}$$

where $\hat{x}$ is the estimated state, y is system output, and $\hat{y}$ is the observer output, and K is the observer gain matrix. Here we design K so that the observer is asymptotically stable. Hence quantitative estimation error; $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$, tends to zero, whenever t tends to ∞; i.e.

$$\|\boldsymbol{e}(t)\| \rightarrow 0 \quad \text{when} \quad t \rightarrow \infty \tag{11}$$

A qualitative model for observer of Eq. (10) is derived by an abstraction algorithm [11]. This qualitative model is described by a N.D.A. which is the core of our qualitative state estimator (c.f. Fig 1.). The obtained qualitative observer has two noticeable differences with the quantitvie state observer: The observer inputs are qualitative signals, and the describing model is a qualitative model.

To investigate the convergence of the estimated qualitative states, we assume that all partitions are rectangular and equal distance. In addition, we define qualitative state convergence as follows.

Definition: The qualitative state Q_i converges to qualitative state Q_j if the distance between two qualitative states; that is denoted by $d_{q_{ij}}(t)$, is uniformly ultimate bounded (U.U.B.). In this definition

$$d_{q_{ij}}(t) \triangleq \left\| M_{Q_i}(t) - M_{Q_j}(t) \right\| \tag{12}$$

where M_{Q_i} and M_{Q_j} are the center points of Q_i and Q_j, respectively. Since it is assumed that all partitions are equal distance, then Eq. (12) is a distance function. In addition, $d_{q_{ij}}(t)$ is U.U.B. if and only if,

$$\exists T \quad, \gamma > 0 \quad \text{s.t.}\ \forall t > T \quad \left\| d_{q_{ij}}(t) \right\| < \gamma \tag{13}$$

Theorem: If estimation error in a quantitative state observer converges to zero asymptotically, then in qualitative observer, the qualitative estimated state converges to the actual qualitative state.

Proof: In qualitative observer (c.f. Fig. 1), inputs are qualitative signals, $[u]$ and $[y]$ respectively. we assume that $\boldsymbol{u}(t)$ is a pure discrete signal. (This is true in many cases such as discretely controlled continuous systems that are found in many technological fields [9]. In addition if $u(t)$ is not discrete, a bounded term will be added in Eq. (14) and therefore the value of δ in Eq. (15) will be changed.) Thus we rewrite the Eq. (11) as follows.

$$\boldsymbol{e}^{\bullet}(t) = (\boldsymbol{A} - \boldsymbol{KC})\boldsymbol{e}(t) + \boldsymbol{K}\,\Delta\boldsymbol{y}(t) \;,\; t \geq 0 \tag{14}$$

where $\Delta\boldsymbol{y}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_{\varphi}(t)$ and $\boldsymbol{y}_{\varphi}(t)$ is output of the injector. For injector we have:

$$\boldsymbol{y}_{\varphi}(t) \in [\boldsymbol{y}(t)] \;\Rightarrow \|\Delta\boldsymbol{y}(t)\| < \delta \tag{15}$$

The quantitative state observer is assumed asymptotically stable. It means that all eigen-values of $(\boldsymbol{A} - \boldsymbol{KC})$ have negative real parts.It is resulted that $\|\boldsymbol{e}(t)\|$ is U.U.B. and then for each initial value $\boldsymbol{e}(0)$,

$$\exists T \;,\alpha > 0 \text{ s.t. } \|\boldsymbol{e}(T)\| < \alpha \;, \text{t} > T \tag{16}$$

T is considered as convergence time of quantized observer and in qualitative abstracting of the observer the sampling time, T_S, must satisfy condition $T \leq T_S$. By this consideration the qualitative observer is built for converged quantitative observer and we are sure that estimation error for obtained qualitative model is bounded. Now by definition 1, qualitative estimation error, $d_{q_{x\hat{x}}}$, is defined as distance between actual qualitative state $[\boldsymbol{x}]$, and estimated qualitative state $[\hat{\boldsymbol{x}}]$; that is,

$$d_{q_{x\hat{x}}} = \left\| M_{\mathrm{Q_x}}(\mathrm{t}) - M_{\mathrm{Q_{\hat{x}}}}(\mathrm{t}) \right\| \tag{17}$$

where $M_{\mathrm{Q_x}}(\mathrm{t})$ and $M_{\mathrm{Q_{\hat{x}}}}(\mathrm{t})$ are center points of $[\boldsymbol{x}]$ and $[\hat{\boldsymbol{x}}]$, respectively. By using triangular property of norms,

$$d_{q_{x\hat{x}}} \leq \left\| M_{\mathrm{Q_x}}(\mathrm{t}) - \boldsymbol{x}(\mathrm{t}) \right\| + \left\| \boldsymbol{x}(\mathrm{t}) - \hat{\boldsymbol{x}}(\mathrm{t}) \right\| + \left\| M_{\mathrm{Q_{\hat{x}}}}(\mathrm{t}) - \hat{\boldsymbol{x}}(\mathrm{t}) \right\|. \tag{18}$$

By considering property of qualitative modeling, it is seen that the first and the third terms in right hand side of Eq. (18) are bounded. Similarly, it is resulted that the second term is also bounded. Therefore $d_{q_{x\hat{x}}}$ is U.U.B and by Eq. (13), the qualitative estimated state converges to the actual qualitative state.

We call convex convergence set of Q.E.E as convergence set. As it is well-known, increasing observer gain matrix **K**, moves poles of observer far from the imaginary axis in s-plane. This fact leads to increasing convergence rate of quantitative observer and reducing convergence time. On the other hand increasing K leads to increasing uncertainty introduced by injector in Eq. (14) and convergence set respectively, that leads to additional estimated states. Thus in designing the quantitative observer, this trade off must be considered.

In next section the proposed qualitative observer is simulated for a two-tank system and some cases are considered for further illustration.

4. Illustrative Example

In the following, the proposed observer is simulated on a two-tank system which is depicted in Fig. 2. The system behavior is described by the following differential equations [11].

$$\begin{aligned}
\frac{d\boldsymbol{h}_1}{d\boldsymbol{t}} &= \frac{1}{\boldsymbol{A}}(\boldsymbol{q}_{\boldsymbol{in}} \cdot \boldsymbol{P}(\boldsymbol{t}) - \boldsymbol{S}_{\boldsymbol{v}} \cdot \mathrm{sgn}(\boldsymbol{h}_1 - \boldsymbol{h}_2) \cdot \sqrt{2\boldsymbol{g}\,|\boldsymbol{h}_1 - \boldsymbol{h}_2|}) \\
\frac{d\boldsymbol{h}_2}{d\boldsymbol{t}} &= \frac{1}{\boldsymbol{A}}(\boldsymbol{S}_{\boldsymbol{v}} \cdot \mathrm{sgn}(\boldsymbol{h}_1 - \boldsymbol{h}_2) \cdot \sqrt{2\boldsymbol{g}\,|\boldsymbol{h}_1 - \boldsymbol{h}_2|} - \boldsymbol{S}_{\boldsymbol{v}} \cdot \sqrt{2\boldsymbol{g}\,|\boldsymbol{h}_2|})
\end{aligned} \tag{19}$$

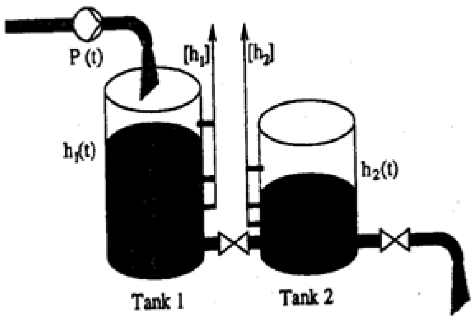

Fig. 2. Two tank system

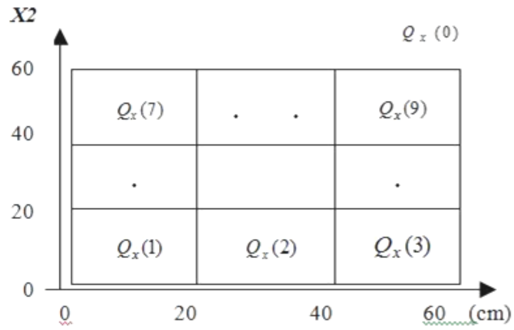

Fig. 3. State partitions of the two tank system.

Where $\boldsymbol{x} = (h_1, h_2)$ is the system state vector and model parameters are given in table 1.

The pump can work in 3 modes, "off", "medium power", and, "full power". Thus the input qualitative values are "off", "medium", and "full ".

For qualitative modeling we use the Quamo Toolbox [12]. In this modeling the continuous system state space is partitioned as shown in Fig. 3. The level of tank 2 is considered as output of the system and consequently with the same partitioning of h_2.

Here the quantitative observer is considered as a Leunberger observer. To investigate the effects of the parameters of quantitative observer and sampling time of qualitative modeling procedure, four cases are considered as follows.

- $T_s = 5_s$ and $\boldsymbol{P} = \{-1, -1\}$
- $T_s = 5_s$ and $\boldsymbol{P} = \{-0.1, -0.1\}$
- $T_s = 10_s$ and $\boldsymbol{P} = \{-1, -1\}$
- $T_s = 10_s$ and $\boldsymbol{P} = \{-0.1, -0.1\}$

Table.1.
Parameters of the tank system

Description	***parameter***	***value***
Cross-section area of each tank	A	$1.5 \cdot 10^{-2}\ m^2$
Cross-section of the valves	S_v	$.00002 m^2$
Input flow if the pomp is on	q_{in}	$4.5 l / \min$
The gravity constant	g	$9.81 m / s^2$

Where T_s is sampling time and $\boldsymbol{P}$ denoted pole of the observer. To simulate these four cases an input sequence is applied to the plant model and corresponding qualitative output and actual qualitative state of the plant is computed. The obtained results are shown in Fig. 4 for $T_s = 5_s$, and, in Fig. 6 for $T_s = 10_s$, respectively.

Estimated states for cases 1 and 2 are depicted in Fig. 5. Comparison of results with the actual states shows that in both cases the observers can estimate qualitative states completely. In addition, for case 1, fewer spurious states are generated rather than pure qualitative observer. But observer of case 2 generated more spurious states than the pure qualitative observer. By comparison observer poles in case 1 and 2, it can be seen that, for case 1 poles are chosen properly so that the convergence time T is less than T_s. But for case 2 convergence time of the observer is increased by choosing $\boldsymbol{P} = \{-0.1, -0.1\}$, so that the observer has not been converged for $T_s = 5_s$. The same investigations are repeated in case 3 and 4 for $T_s = 10_s$ (c.f. Fig. 7). The obtained results show that all observers are not complete, (transient state in 50s cannot be estimated correctly by all observers). This is due to increasing T_s, that leads to loss of transient modes of the main system and quantitative observer in obtained qualitative models. In fact, changing the sampling time is not a good solution to satisfy convergence time condition, and in many actual cases, it is impossible as well. By comparison Figs. 6-c and 7-c, it is seen that the observer of case 4 has generated less spurious states than the observer of case 2, because the convergence time condition is satisfied better than for case 4.

By considering these four cases, it is resulted that convergence time condition must be satisfied by adjusting parameters of quantitative observer and in the obtained qualitative observer must be complete. A good experimental criterion that can be used is

$$\frac{T_s}{10} \le T \le \frac{T_s}{5} \tag{20}$$

5. Conclusion

In this paper a method is presented to design a qualitative state observer. Our method uses a quantitative observer as the core of designed qualitative observer. By this method, we are able to use the advantages of quantitative observer deign methods; such as flexibility in convergence time. In addition, the necessary signals can be measured qualitatively. The main result of the paper presents the coefficient conditions for existence of the proposed observer. The simulation results show that adjusting parameters of quantitative observer leads to reduction of spurious states and time of state estimation that both are important in some applications such as fault diagnosis systems. Using nonlinear quantitative observers and refining of state space-output space partitions are interesting topics that can be studied in the feature works.

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